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SCALABLE ALGORITHM FOR NON-LINEAR PROBLEMS OF SOLID MECHANICS

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Abstract: *The paper is concerned with a novel algorithm for solution to contact problems stemming from the TFETI (Total Finite Element Tearing and Interconnecting) domain decomposition method. The TFETI based technique converts the original contact problem to the quadratic programming one with the equalities and simple bound constraints. Our new algorithm exhibits both parallel and numerical scalabilities so that it enables us to effectively solve steady-state problems of deformable bodies undergoing contact, geometric and material non-linear effects. In this paper we propose algorithm with nested iteration strategy, where its inner part consists of a new version of our previously developed MPRGP and SMALBE algorithms and the outer loop iterates on the geometric and material non-linearities. Numerical experiments include solutions to steady-state problems with non-linear effects. .*

Keywords: ***Contact non-linearity, Geometric non-linearity, Material non-linearity, Domain decomposition, Scalability.***

The paper is concerned with a novel algorithm for solution to contact problems stemming from the TFETI (Total Finite Element Tearing and Interconnecting) domain decomposition method. The TFETI method is based on idea that the compatibility between non-overlapping sub-domains, into which the original domain is partitioned, is enforced by the Lagrange multipliers. The distinctive feature of the TFETI consists in the fact that the method also enforces the Dirichlet boundary conditions by means of the Lagrange multipliers. The TFETI based technique converts the original contact problem to the quadratic programming one with the equalities and simple bound constraints. Moreover, it also results in more efficient preconditioning by an enriched natural coarse grid defined by a priory known kernels of the stiffness matrices. Our new algorithm exhibits both parallel and numerical scalabilities so that it enables us to effectively solve steady-state problems of deformable bodies undergoing contact, geometric and material non-linear effects. In this paper we propose algorithm with nested iteration strategy, where its inner part consists of a new version of our previously developed MPRGP and SMALBE algorithms and the outer loop iterates on the geometric and material non-linearities. Numerical experiments include solutions to steady-state problems with non-linear effects and their results document that the proposed algorithms are robust, highly accurate and exhibit both parallel and numerical scalabilities.

Dostál et al.(2011) analysed problem of frictionless contact problem between solid bodies, while they considered both geometrically and materially linear cases. Therein they suggested a new in a sense optimal version of their own previously developed algorithm based on TFETI domain decomposition method. The goal of this paper is to apply this new algorithm to the contact problems accompanied by both geometric and material non-linear phenomena, and to show that it can yield, even under these conditions, good results.

The FETI domain decomposition method was introduced by Farhat and Roux (1991) as a parallel finite element solver for the self-adjoint elliptic partial differential equations. Its key idea is a decomposition of the spatial domain into non-overlapping sub-domains that are 'glued' by Lagrange multipliers, so that, after eliminating the primal variables, or displacements, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is to be solved iteratively.

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The partition of the original domain into sub-domains usually generates some ‘floating’ sub-domains with not enough prescribed displacements, so that their stiffness matrices are singular in steady-state cases and implementation of FETI then includes the computation of their kernels. However, stable evaluation of the bases of the kernels, though theoretically clear in exact arithmetic context, is tricky in the presence of the round-off errors. To overcome this difficulty, Dostál et al. (2006) suggested enforcement of all the Dirichlet boundary conditions by the Lagrange multipliers so that all the sub-domains were treated as totally unconstrained. This version of FETI is referred to as the Total FETI (TFETI). Since the kernels of stiffness matrices of all the sub-domains are the same and known beforehand, this approach removed the problems with identification of these kernels.

Even though the FETI class methods were originally developed for numerical solution to linear elliptic partial differential equations, it turned out that they were even more successful for the solution to contact problems.

We show that application of the TFETI methodology to the contact problems converts the original problem to the quadratic programming problem with bound and equality constraints and well-conditioned regular part of the Hessian matrix. Such problems are to be solved very efficiently by the recently proposed algorithms. The problem is first reduced by Semi-Monotonic Augmented Lagrangians with Bound and Equality constraints (SMALBE) method to the sequence of bound constrained quadratic programming problems. SMALBE accepts inexact solutions of auxiliary bound constrained problems solved approximately in the inner loop until the norm of the projected gradient is proportional to the feasibility error, and updates the regularisation parameter until the value of the Lagrangian increases. These auxiliary problems are to be solved efficiently by the Modified Proportioning with Reduced Gradient Projection (MPRGP) method in the inner loop. A unique feature of MPRGP is the rate of convergence which is independent of the inequality constraints.

The performance of a domain decomposition based iterative method depends on two important properties, namely numerical and parallel scalabilities. Such a method is said to be numerically scalable if the condition number of the problem does not grow or grows weakly with the ratio of the sub-domain size and the mesh size. The parallel scalability represents ability of an algorithm to achieve larger speed-ups for a larger number of processes.

The primary interest of this work is the development of effective strategy for fully non-linear problems, where, in addition to the contact interaction, the kinematics of the body system are not confined to small strains, and where the material response is potentially non-linear and inelastic.

The paper presents results of two sets of numerical experiments with the proposed algorithms. The first one is concerned with analysis of a bolt and nut contact problem and the second one shows results concerning the numerical and parallel scalabilities. All the numerical experiments were carried out with our in-house general purpose finite element package PMD (Package for Machine Design) [PMD].

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