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B-SPLINE FINITE ELEMENT METHOD IN ONE-DIMENSIONAL ELASTIC WAVE PROPAGATION PROBLEMS

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Abstract: *In this paper, the spline variant of finite element method (FEM) is tested in one-dimensional elastic wave propagation problems. The special attention is paid to propagation of stress discontinuities as an outcome of the shock loading and also to spurious oscillations occurring near theoretical wave-fronts. Spline variant of FEM is a modern strategy for numerical solution of partial differential equations. This method is based on spline basic functions as shape, testing functions in FEM content. For examples, B-splines, T-splines, NURBS and more others could be applied. For one-dimensional problems, B-spline representation is sufficient. B-spline basis functions are piecewise polynomial functions. It was shown, that B-spline shape functions produce outstanding convergence and dispersion properties and also appropriate frequency errors in elastodynamics problems. In this initial work, accuracy, convergence and stability of the B-spline based FEM are studied in numerical modelling of one-dimensional elastic wave propagation of stress discontinuities. For the time integration, the Newmark method, the central difference method and the generalized-α method are employed.*

Keywords: ***elastic wave propagation, B-spline based finite element method, spurious oscillations.***

1. Introduction

A modern approach in the finite element analysis is the isogeometric analysis (IGA), see Cottrell et al. (2009), where shape functions are based on varied types of splines. This approach has an advantage that the geometry and approximation of the field of unknown quantities is prescribed by the same technique. Another benefit is that the approximation is smooth. It was shown for the IGA approach, that the optical modes did not exist unlike higher-order Lagrangian finite elements, see Cottrell et al. (2006); Hughes et al. (2008). Further, dispersion and frequency errors for the isogeometric analysis were reported to decrease with the increasing order of spline, see Cottrell et al. (2009). The spline based FEM with the small dispersion errors and the variation diminishing property, see Piegl and Tiller (1997), could eliminate the spurious oscillations, which are the outcome of the Gibb's effect and dispersion behaviour of FEM, see Chin (1975) and Belytschko and Mullen (1978).

2. Finite element response

In this paper, the semidiscretization method is tested in one-dimensional elastic wave propagation of sharp wave fronts and stress discontinuities. For the spatial discretization, the continuous Galerkin's approximation method is employed, see Hughes (1983). For the time integration, the Newmark method, see Newmark (1959), the central difference method, see Dokainish and Subbaraj (1989), and the implicit form of the generalized-α method, see Chung and Hulbert (1993), are employed.

The bar is discretized by linear and cubic B-splines with $N = 101$ control points. For the linear B-spline discretization, the knot vector is used uniform, see Piegl and Tiller (1997), and the control points are distributed uniformly with constant distances. For the cubic B-spline discretization, the knot vector is also employed uniform, but the control points are given by the Greville abscissa, see Greville (1967). For example, stress waveforms at time $t = 0.5L/c_0$ computed by the implicit generalized-α method with spectral radius $\rho = 0.5$ are presented on Fig. 1. c_0 is the wave speed in an elastic bar, L is the length of a bar. The theoretical wavefront takes place in half of the bar and the stress value in the overlaying should hold the magnitude $\sigma = -\sigma_0$.

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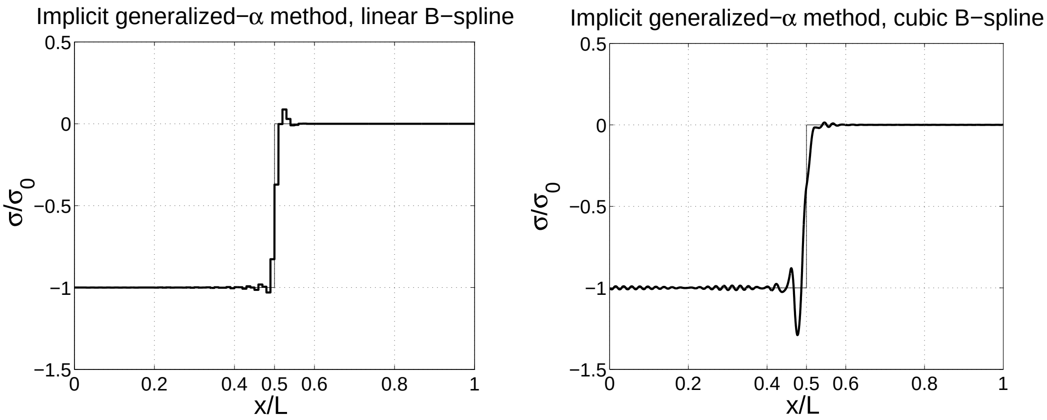

Fig. 1: Stress in an elastic bar under the shock loading at time $t = 0.5L/c_0$ computed by the implicit generalized-α method with spectral radius $\rho_\infty = 0.5$ for linear (left) and cubic (right) B-splines.

3. Discussion and conclusions

In the numerical test of stress discontinuity propagation problem computed by B-spline variant of FEM, the oscillations near sharp wavefronts are smaller than for the classical FEM due to the variation diminishing property and smaller dispersion errors. The post-shock oscillations are typical for the central difference method due to the 'row sum' diagonal mass matrix. This diagonal mass matrix is only of second order and also it produces unsuitable frequency spectrum. On the other side, the Newmark method and the implicit form of the generalized-α method produce the both types of oscillations, both post-shock and front-shock oscillations. Jumps in behaviour of a stress function obtained by the implicit form of the generalized-α method with high level of frequency dissipation are well approximated. Nevertheless, the total energy is not preserved.

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