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LOCAL INTEGRAL FORMULATIONS FOR THIN PLATE BENDING PROBLEMS

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Abstract: *It is well known that high order derivatives of field variables in the governing equations give rise to difficulties in solution of boundary value problems because of worse accuracy of numerically evaluated high order derivatives. The order of the differential operator can be decreased by decomposing this operator into two lower order differential operators with introducing new field variable.*

In two recent decades, solution of many engineering problems as well as problems of mathematical physics have been reformulated by using various mesh free formulations with meshless approximations. In this paper, we present that the decomposition of the biharmonic equation into two Poisson equations is applicable to general case of boundary conditions and any shape of the boundary edge of the plate, if we use the Local Integral Equation (LIE) formulation and a meshless approximation for primary field variables. Besides the standard advantages of mesh free formulations remember the new advantage consisting in decreasing the order of the derivatives of field variables. Instead of the third order derivatives of the deflection field in the formulation for the biharmonic equation the highest order of the derivatives in the present formulation does not exceed the first order in cases of clamped and/or simply supported edges while the second order derivatives are required in the case of free edges on plates. Several illustrative examples will be presented for comparison of accuracy, convergence and computational efficiency achieved by using various approaches.

Keywords: ***Local Integral Equation formulation, meshless approximation, decomposition, biharmonic equation, Poisson equations***

1. Introduction

In this paper, we present that the decomposition of the biharmonic equation into two Poisson equations is applicable to general case of boundary conditions and any shape of the boundary edge of the plate, if we use either the strong formulation or the Local Integral Equation (LIE) formulation and a meshless approximation for primary field variables. In the local weak formulation, the constant test function with the support on the sub-domain is utilized, what corresponds to integral satisfaction of physical balance principles on local sub-domains. Owing to the decomposition, the order of the derivatives in the governing equations is decreased from four to two. Two kinds of the meshless approximations are employed in this paper, such as the Moving Least Square (MLS) approximation (Lancaster and Salkauskas, 1981) and the Point Interpolation Method (PIM) (Liu, 2003).

2. Physical decomposition of governing equations

For the plate of thickness h and midplane Ω orthogonal to the axis x_3, the tensor of moments can be expressed in terms of the second order derivative of deflection as

$$M_{ij} = -D\left[(1-\nu)w_{,ij} + \nu\delta_{ij}\nabla^2 w\right], \quad D = \frac{Eh^3}{12(1-\nu^2)} \tag{1}$$

where D is the bending stiffness, E and ν is the Young modulus and Poisson ratio, respectively.
The governing equation for deflections of thin plane is given as

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$$M_{ij,ij}(\mathbf{x}) = -q(\mathbf{x}), \tag{2}$$

If we shall consider the bending stiffness to be constant, after substituting (1) to (2) we can obtain simplified governing equations in form

$$D\nabla^2\nabla^2 w = q \tag{3}$$

The fourth order derivatives of deflections in governing equations can give rise to serious difficulties not only in strong formulation for numerical solution, but also in weak formulation owing to inaccurate approximation of high order derivatives of deflections occurring in the integral equations.

Therefore, it is expedient to introduce the new field variable defined as

$$m(\mathbf{x}) := -D\nabla^2 w(\mathbf{x}) \quad \text{for} \quad \mathbf{x}\in\Omega \tag{4}$$

Then the governing equation (3) is split into two equations given by (4) and (5)

$$\nabla^2 m(\mathbf{x}) = q(\mathbf{x}) \quad \text{for..} \quad \mathbf{x}\in\Omega \tag{5}$$

3. Conclusions

The decomposition of the thin plate bending problems governed by the biharmonic operator into two coupled problems governed by Poisson equations is developed and discussed. Two kinds of meshless approximations of field variables are employed in each formulation for numerical solution of boundary value problems. The developed decomposed formulations are not restricted to certain class of boundary value problems. For solution of the decomposed problem, both the strong and local weak formulations have been developed. The accuracy, convergence of accuracy and computational efficiency have been studied for two formulations combined with two meshless approximation of field variables in simple boundary value problems for circular plate. The discussed methods give reasonable numerical results when applied to decomposed problem, while the methods applied to original biharmonic problem fail.

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