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# BENDING OF FUNCTIONALLY GRADED CIRCULAR PLATES WITH PIEZOELECTRIC LAYER BY THE MLPG METHOD

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**Abstract:** *A meshless local Petrov-Galerkin (MLPG) method is applied to solve bending of circular plate with piezoelectric layer attached at the top. Plate is analyzed as a 3D axisymmetric. Functionally graded material properties with continuous variation in the plate thickness direction are considered. Piezoelectric layer with applied nonzero voltage difference acts as a piezoelectric actuator, thus deflection of the plate can be controlled. Local integral equations are defined from the set of governing equations for mechanical and electric fields using appropriate test functions. Spatial variation of all physical fields is approximated by the moving least-squares (MLS) method only in terms of nodes. After performing all spatial integrations the system of ordinary differential equations is finally obtained and solved by Houbolt finite-difference scheme.*

**Keywords:** ***Meshless local Petrov-Galerkin method (MLPG), moving least-squares (MLS) approximation, piezoelectric actuation, functionally graded materials.***

## 1. Introduction

Advanced structural systems are required to be low-weight, high-strength and often to have also self-monitoring capabilities. Recent progress in engineering and material sciences offers new possibility in design of such structures; the multifunctional composites (Gibson, 2010) composed of so-called smart materials. Among many smart materials the piezoelectric materials are dominantly used for control and suppression of structural vibration (Adachi et al., 1994) because of their sensory/active capabilities. In the recent years also the functionally graded materials (FGMs) (Suresh and Mortensen, 1998) are widely applied in structural design because of their excellent properties. FGMs are multi-component composite materials in which the volume fraction of the material constituents is varying in a predominant direction. This feature can be used to tune the selected properties into desired value. For example structural element can be designed to have the strength of steel on one side combined with the heat resistance of ceramics on the other side.

Analysis of complex structural systems requires advanced numerical methods because of complex geometry or boundary conditions. Although the well established finite element method (FEM) is applicable to analysis of piezoelectric structures (Benjeddou, 2000), the analysis of materials with continuously nonhomogeneous properties such as FGMs can lead to certain difficulties. The material coefficients in commercial FEM codes are assumed to be constant within an element, thus leading to piecewise homogeneous idealization of FGMs. In the last decade, an increasing attention has been devoted to meshfree or meshless methods for numerical analyses. The motivation is clear from their name; to avoid difficulties associated with mesh of finite elements such as expensive mesh generation, shear locking or above mentioned difficulties in modeling of continuously nonhomogeneous media. The meshless local Petrov-Galerkin (MLPG) method (Atluri, 2004) is considered as a basis for many meshless methods. Meshless formulations based on the MLPG were recently applied to laminated plates (Sladek et al, 2010a) and also to piezoelectric plates (Sladek et al, 2010b).

In the present paper the analysis of functionally graded circular plate with piezoelectric actuator is presented as shown in Fig. 1a. Homogeneous material properties are considered in the actuator. A circular plate (having index 1 in Fig.1) together with piezoelectric actuator (index 2) can be considered as a 3-D axisymmetric body with axis of symmetry passing through the center of the plate. With use of

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cylindrical coordinates the original 3-D axisymmetric problem can be reduced to 2-D problem considered on the cross-section of the plate (see Fig. 1b). An exponential variation of material properties is assumed for the FGM plate. The coupled electro-mechanical fields are described by constitutive relations and governing partial differential equations (PDEs). Nodal points are spread on the analyzed domain without any restrictions. Small local circular subdomain is introduced around each nodal point. Local integral equations (LIEs) constructed from governing PDEs are defined over these circular subdomains. For a simple shape of subdomains- like circles used here, numerical integration of LIEs can be easily carried out. Moving Least-Squares (MLS) approximation scheme (Lancaster & Salkauskas, 1981) is used to approximate the spatial variations of electric and mechanical fields. MLS scheme ensures $C^1$ continuity in each layer, but not across the material interface of plate and piezoelectric actuator. Thus MLS approximation is carried out separately in each considered layer. Additional coupling equations are considered for nodes on the interface to ensure the continuity of primary variables and the equilibrium of the tractions and electric charge. The essential boundary conditions are satisfied by the collocation of MLS approximation expressions for unknowns at boundary nodes. After performing the MLS approximation a system of ordinary differential equations (ODEs) for certain nodal unknowns is obtained. The effect of material gradation and deflection suppression by active piezoelectric layer are observed on several numerical examples.

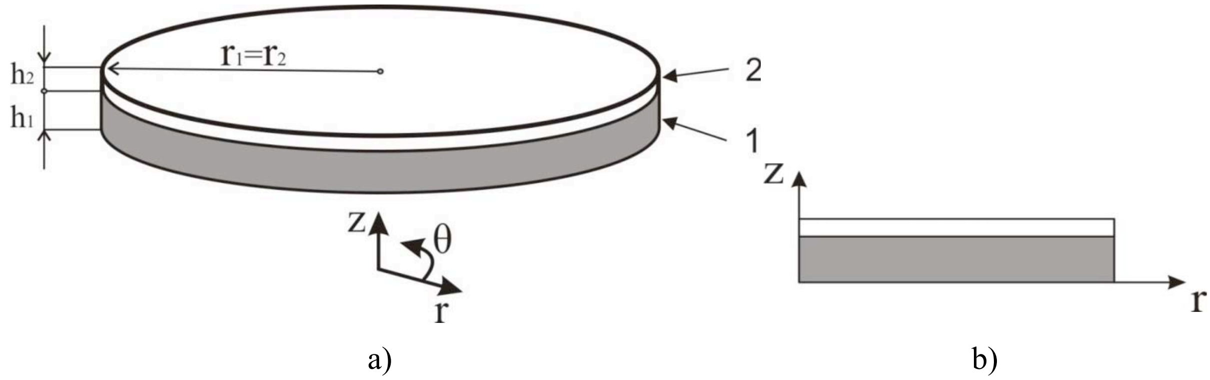


*Fig. 1: Geometry of the circular plate: a) original 3-D problem, b) assumed 2-D geometry*

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