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TWO-SCALE MODELLING OF STRONGLY HETEROGENEOUS CONTINUA USING THE HOMOGENIZATION APPROACH

E. Rohan* **R. Cimrman**** **V. Lukeš*****

Abstract: *The notion of strong heterogeneity is considered in the sense of material scaling: the idea is to study mathematical models where the coefficients of the partial differential equations associated with one of the material phases depend on the characteristic size ε of the microstructure. This modeling ansatz is justified to represent high contrasts in material properties of different components; it was applied to study wave propagation in two-phase elastic composites with "weak" inclusions, where elasticity is scaled by ε^2, or to describe poroelastic behaviour in double-porous media, where permeability of the second porosity is proportional ε^2. Perforated structures can be handled using similar mathematical tools. For homogenization of thin structures the scaling is related to the thickness, which leads to reduced spatial dimension of the problem. This paper summarizes some models developed using the homogenization approach; namely applications in modelling elastic waves, acoustic transmission and fluid flow in porous media are discussed.*

Keywords: ***homogenization, composites, wave propagation, porous media, perfusion***

1. Introduction

Homogenization techniques were developed to *simplify* description and modeling of heterogeneous structures where the characteristic size of the heterogeneities is much smaller than the size of the whole structure. Usually the *simplification* is associated with tractability of numerical modeling: "original structures", like fiber composites, or masonry are constructed by repeating some heterogeneity patterns which are related to oscillation in material coefficients. Therefore, the "direct modeling approach", where all the oscillations must be captured, may lead to extremely large "discretized" models with untractable numbers of equations to be solved. The homogenization-based analysis and numerical modeling is based on computing effective material properties characterizing the heterogeneity patterns. Thus, in contrast with the "original" models with oscillating material parameters, the "homogenized models" involve (locally) "constant" effective material coefficients, so that the number of equations obtained by the discretization is reduced by several orders. Information about the geometry and topology of the microstructure is transformed into the effective material coefficients by means of solving so-called "microscopic problems".

When more complex heterogeneous continua are considered, where *qualitatively* different phases interact at the "microscopic level", the homogenization results in new models which differ in their structure from any of the models describing the particular phases. In this way, *new constitutive laws* can be obtained, characterized by effective material parameters with very clear physical explanation. This is a great advantage of the homogenization-based modeling which cannot be achieved easily by phenomenological approaches.

In the paper we focus on selected topics where the homogenization approach proves to be very efficient in the sense just explained: two major themes are considered 1) wave propagation in strongly heterogeneous solids, 2) perfusion in double-porous fluid-saturated media. The following examples were treated by the homogenization method:

* Prof. Dr. Ing. Eduard Rohan: Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Plzeň; CZ, e-mail: rohan@kme.zcu.cz
** Ing. Robert Cimrman, Ph.D.: New Technologies Research Centre, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Plzeň; CZ, e-mail: cimrman3@ntc.zcu.cz
*** Ing. Vladimír Lukeš, Ph.D.: Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Plzeň; CZ, e-mail: lukes@kme.zcu.cz

- waves in phononic crystals – elastic, or piezo-elastic high contrast composites, Ávila et al. (2008); Rohan et al. (2009); Leugering et al. (2010),
- acoustic waves and transmission on thin (rigid) sieves, acoustic fluid in layers containing obstacles, Rohan and Lukeš (2010); Leugering et al. (2010),
- Biot compressible medium – fluid saturated porous material (FSPM) with double porosity, two-compartment and three-compartment models, application to bone poroelasticity and perfusion in deforming tissues, Rohan and Cimrman (2011); Rohan et al. (2012); Rohan and Cimrman (2010),
- Darcy flow in double-porous layer, application to perfusion, Rohan (2010).

Although all these problems are linear, also for nonlinear problems a linearization in terms of incremental updating schemes can be proposed, see e.g. Rohan (2006), to allow for decoupling the so-called limit two-scale problems into the macroscopic and microscopic ones. The homogenization approach provides computationally efficient schemes for the multi-scale modeling. Once the macroscopic response is obtained, the "microscopic" responses can be reconstructed using the characteristic local responses. There is a remarkable difference with respect to the standard homogenization in using such schemes: while in a standard case the homogenization result is really independent of the heterogeneity scale, in the "large contrast" case the real material coefficients are defined for a given scale $\varepsilon_0 > 0$. This means that the limit model must be interpreted by an extrapolation for the scale ε_0, whereby the so-called corrector result is used.

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