18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 284–285
Paper #122

EMASS LUMPING METHODS FOR THE SEMI-LOOF SHELL ELEMENT

V. Sháněl*, R. Kolman, J. Plešek*****

Abstract: *Mass matrix diagonalization in terms of a finite element method (FEM) is essential for an effective deployment of the explicit method as one of the direct integration methods of the motion equations of elastodynamics. A particular attention is focused on the mass matrix diagonalization of the semi-loof shell element. Its diagonalization requires a specially designed universal diagonalization scheme that is derived from the scaling HRZ method. Another analyzed aspect is the problem of preserving the moment of inertia for various types of finite elements. The proposed scheme is implemented in the finite element program and consequently tested on several problems.*

Keywords: ***FEM, mass matrix lumping, semi-loof finite shell element, impacts and waves in solids, elastodynamics.***

1. Introduction

The purpose of this paper is a study of mass matrix diagonalization in terms of a finite element method (FEM). It is essential for an effective deployment of the explicit method as one of the direct integration methods of the motion equations of elastodynamics (Hughes 1987). Explicit methods are ideal for investigating very short and fast processes, for example wave propagation and nonlinear problems. If a lumped mass matrix is used, the motion equation falls apart into separately solvable equations, thence no matrix equation solver is needed and the cost per time step is small. Lumping is not a unique process. There are several algorithms which generally produce different results with various properties. In the past, these processes were based on purely physical concepts. Nowadays, there are three different main approaches. Among the most important characteristics of the methods belong positive definiteness of the mass matrix, the preservation of the element mass, the position of a gravity center, inertia and the necessity to compose a consistent mass matrix. All these conditions cannot be met in general. In the dynamics application, it was observed that any lumping process that preserves the element mass will lead to convergence (Cook et al. 2002).

The diagonalization is performed as universal for various types of finite elements containing rotation and translation degrees of freedom. A particular attention is focused on the mass matrix diagonalization of the semi-loof shell element (Sháněl 2011). This element has been widely used for elastic analysis and later extended for material and geometric nonlinearities. The element is available in the 8-node quadrilateral and 6-node triangle form which significantly aids the meshing process. Its diagonalization requires a specially designed universal diagonalization scheme that is derived from the scaling HRZ method (Hinton et al. 1976). This scheme has a plenty of advantages compared to other lumping techniques. Mainly, the HRZ scheme is able to diagonalize mass matrices of the elements which include rotational as well as translational DOFs. A modification of the original procedure which ensures proper rotational DOFs scaling is discussed. Another advantage of this scheme is that an assemblage of the whole consistent matrix is not necessary - only diagonal terms are required and the implementation is

*Ing. Vít Sháněl: Institute of Thermomechanics AS CR, Impact and Waves in Solids, Dolejškova 1402/5, Prague, CZ, email: shanel@it.cas.cz
**Ing. Radek Kolman, Ph.D.: Institute of Thermomechanics AS CR, Impact and Waves in Solids, Dolejškova 1402/5, Prague, CZ, email: kolman@it.cas.cz
***Ing. Jiří Plešek, CSc.: Institute of Thermomechanics AS CR, Impact and Waves in Solids, Dolejškova 1402/5, Prague, CZ, email: plesek@it.cas.cz

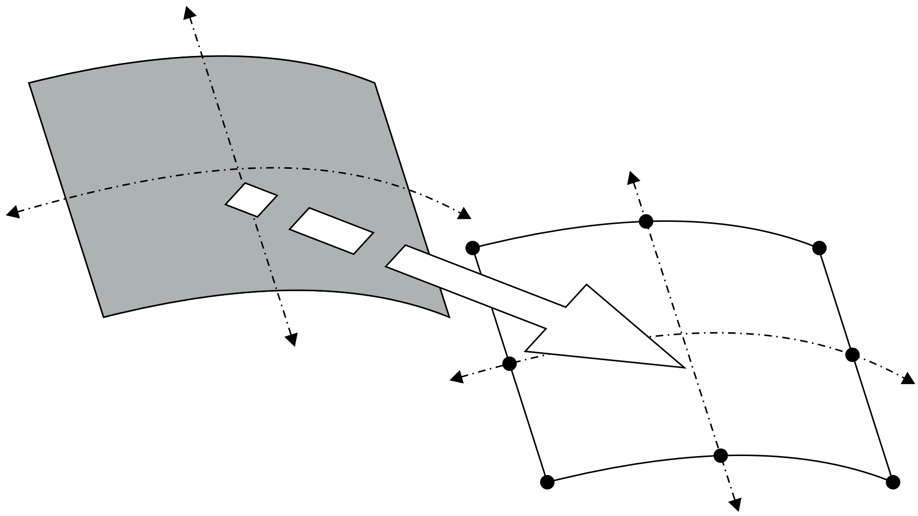

Fig. 1: The mass lumping process

performed on an element level. Another analyzed aspect is the problem of preserving the moment of inertia for various types of finite elements.

2. Conclusions

The standard HRZ method implementation involves a scaling of the rotational degrees of freedom in the same way which applies to the translational degrees of freedom, and it has no physical basis, thus it does not provide any solution for semi-loof shell elements in this form. This was the reason why the interest shifted entirely on the problem of a mass matrix diagonalization with rotational degrees of freedom in which we determined the influence of the rotational degrees of freedom on the total moment of inertia of the semi-loof element. The numerical test proved that the influence of the rotational degrees of freedom was negligible. And hence the implemented algorithm was designed to leave the original values belonging to the rotational degrees of freedom on the diagonal without any changes whatsoever. The proposed scheme is implemented in the finite element program PMD and consequently tested on several problems. The basic verification tests also include computations of natural frequencies of a truss, a beam and an industrial apparatus. This algorithm is further examined on the problem of a head-on impact of two thick plates to provide data about the accuracy of the numeric solution as applied to the elastic wave propagation.

Acknowledgments

This work was supported by the grant projects GA CR 101/09/1630, ME10114, P101/10/P376 and P101/11/0288 under AV0Z20760514.

References

Cook, R. D., Malkus, D. S. & Plesha, M. E. (2002) *Concepts and applications of finite element analysis.* Wiley, New York.

Hinton, E., Rock, T. & Zienkiewicz, O. (1976) A note on mass lumping and related processes in the Finite element method. *Earthquake Engineering and Structural Dynamics*, 13, 9, p. A112.

Hughes, T. J. R. (1987) *The Finite Element Method*, Prentice-Hall Inc., New Jersey.

Sháněl, V. (2011) *On the Mass Lumping in the Finite Element Method.* Master Thesis, CTU in Prague, Faculty of Mechanical Engineering, Prague.