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COMPARISON OF TWO POSSIBLE APPROACHES TO INVERSE LAPLACE TRANSFORM APPLIED TO WAVE PROBLEMS

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Abstract: *When the method of integral transforms is used for solving the system of PDEs describing a wave problem solved, one has to overcome the problem of inverse transform. This work focuses on two possible approaches to the inverse Laplace transform. Using the existing analytical solution of the problem, the classic method making use of the residue theorem and the method based on the numerical inverse Laplace transform are compared. Advantages and disadvantages of both approaches, mainly from computational point of view, are discussed and demonstrated.*

Keywords: ***thin disc, radial impact, non-stationary wave problem, analytical solution, inverse Laplace transform.***

1. Introduction

The application of Fourier method in combination with appropriate integral transform represents a classic method for solving the system of PDEs describing a wave problem solved. In such cases, the inverse integral transform is one of the primary tasks of the process of analytical solution evaluation. Laplace transform represents one of the most used transform in time domain. There exist two possible approaches to its inversion, analytical and numerical. The first mentioned methods are based on the exact evaluation of Bromwich integral defining the inverse Laplace transform (ILT). This is usually done by the help of Cauchy's residue theorem. The methods making use of the numerical evaluation of ILT integral are more powerful and can be applied to more complicated problems. There exist more than one hundred algorithms for numerical inverse Laplace transform (NILT), from simple ones to more sophisticated procedures which usually include sequence accelerators to improve the convergence of numerical process (see Cohen (2007)).

The main aim of this work is to show the possibility of the application of FFT-based algorithm and Wynn's epsilon accelerator (Brančík (1999)) in solving process of a specific wave problem without the lost of analytical results accuracy and to demonstrate the robustness and the efficiency of this numerical approach. In particular, the problem of an elastic thin disc under radial impact is chosen for this purpose. The exact analytical solution can be found in Brepta and Červ (1978).

2. Comparison of analytical and numerical approach to ILT

The analytical approach making use of residue theorem and the numerical method based on FFT and Wynn's epsilon algorithm have been confronted by the help of radial velocity $v_r(r, \varphi, t)$ evaluation at the disc rim ($r = r_1$) and for specific values of angular coordinate φ. The evaluation of analytical solution in time and transform domain has been done for following material and geometric parameters: $r_1 = 0.05\mathrm{m}$, $\rho = 7800\mathrm{kg\,m^{-3}}$, $\nu = 0.3$ and $E = 2.07 \cdot 10^{11}\mathrm{Pa}$. The disc response to radial load specified by the amplitude $\sigma_0 = 1\mathrm{Pa}$ and by the angle $\alpha_0 = \pi/60$ have been studied in time interval $t \in \langle 0, 50\rangle$ μs with constant step $\Delta t = 0.05\mu$s. Dimensionless plots of v_r time histories in selected points are depicted in

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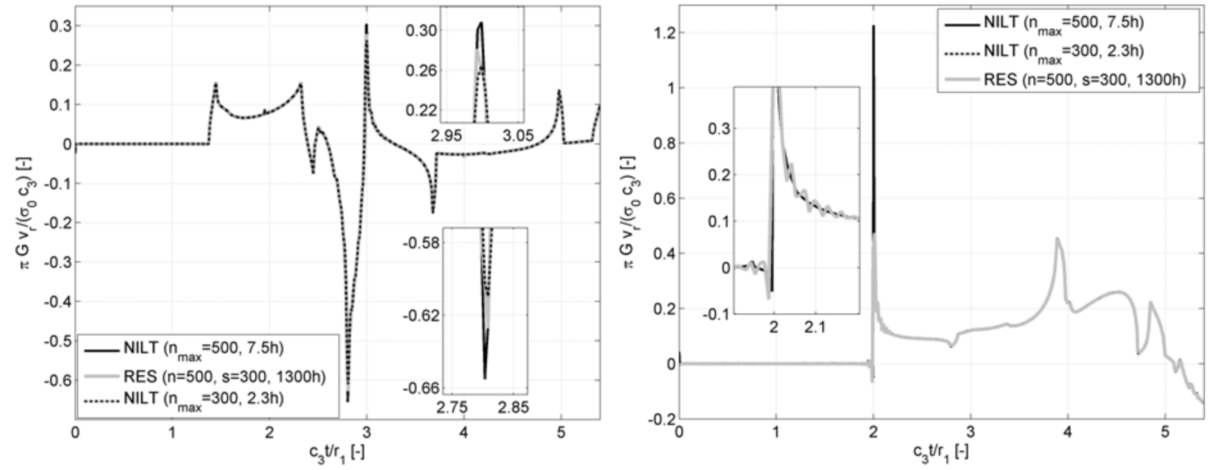

(a) (b)

Fig. 1: Comparison of results obtained by analytical ILT and by NILT: (a) $\varphi = \frac{\pi}{2}$, (b) $\varphi = \pi$

Fig. 1. This figure shows the most accurate results obtained using the analytical ILT approach (curves *RES*), i.e. for the maximal summation indices $n = 500$ and $s = 300$, with those resulted from the application of chosen NILT algorithm (curves *NILT*) and of a comparable accuracy, i.e. for the maximal summation index $n_{max} = 500$ and $n_{max} = 300$. It is evident that the dimensionless time plots of v_r are in good agreement both for $\varphi = \frac{\pi}{2}$ and for $\varphi = \pi$ in the whole time interval of interest. The main discrepancies occur in times corresponding to dominant peaks of v_r, as obvious from detailed views in Fig. 1(a) and from Fig. 1(b). The curves representing results of NILT coincide in the last mentioned figure. Additionally, the results of NILT are probably of higher accuracy, which is indicated by slightly oscillating character of curve *RES* in the vicinity of dominant peak, see the detail in Fig. 1(b).

3. Conclusions

To summary the results of this work we can say that the analytical method making use of residue theorem, contrary to the NILT based method, gives the insight into the physical meaning of each term of the analytical solution in time domain. But its numerical implementation is much more time demanding compared to the obtaining results of the same accuracy by the help of chosen NILT procedure (see the total CPU times stated in the legends of Fig. 1). Another significant advantage of the second mentioned method lies in the possibility of its application to the larger set of problems. Since this approach requires the knowledge of the solution in transform domain only, it can be used for more complicated wave problems, from geometrical, material and boundary/initial conditions point of view, as documented e.g. in Adámek and Valeš (2011). But this method should be used cautiously because there does not exist any universal NILT algorithm suitable for arbitrary problem so the verification of correctness and accuracy of obtained results by another method is important.

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