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COMPARISON OF IMPLICIT-GRADIENT DAMAGE-PLASTIC MODELS

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Abstract: *Damage mechanics coupled with the theory of plasticity is a suitable framework for description of the complex behavior of materials such as concrete, steel, or bone. However, the classical theory fails after the loss of ellipticity of the governing differential equation. From the numerical point of view, loss of ellipticity is manifested by the pathological dependence of the results on the size and orientation of the finite elements. This paper describes two different formulations of coupled damage-plastic models, and their nonlocal enhancements based on the implicit gradient approach. The difference between the formulations is discussed and illustrated by a numerical example.*

Keywords: ***damage, plasticity, nonlocal continuum, implicit-gradient formulation***

1. Introduction

This paper presents coupled damage-plasticity models. Continuum damage mechanics is suitable for the description of stiffness degradation due to the growth of defects such as micro-voids and micro-cracks, while plasticity theory describes permanent deformations of a material induced e.g. by slip mechanisms. However, standard damage-plasticity models with softening would lead to a pathological sensitivity of the numerical solution, converging to physically meaningless results. In this contribution, two different ways of coupling damage with plasticity are considered, and a method that can provide an objective description of localized inelastic processes is described.

2. Plasticity

The main feature of plasticity models is irreversibility of plastic strain. We restrict our attention to the associative plasticity with isotropic hardening or softening under small strain. In the subsequent chapters, we will use the Mises yield condition, which belongs to the most used yield criteria. Note that for Mises plasticity, yielding has a purely deviatoric character.

To implement the constitutive model into a displacement-driven finite element code, an algorithm for the evaluation of the stress increment from a given strain increment must be developed. This procedure is usually called the stress-return algorithm. The stress return algorithm is based on the elastic-plastic operator split, which consists of a trial elastic predictor followed by the return mapping algorithm.

3. Coupling of damage and plasticity

In this section, a brief description of the continuum damage mechanics and its coupling with the plasticity theory is discussed. The isotropic damage mechanics is considered, which means that one single scalar damage variable is introduced. The damage variable describes the reduction of stiffness and strength of material due to the creation, coalescence and growth of voids and microcracks. There exists at least two ways of coupling the plasticity theory to the damage mechanics. The first approach is based on the formulation of the plasticity problem in the effective (i.e. undamaged) stress space. The second

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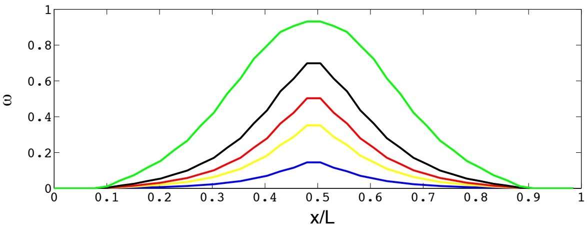

Fig. 1: Evolution of damage profile for formulation 1

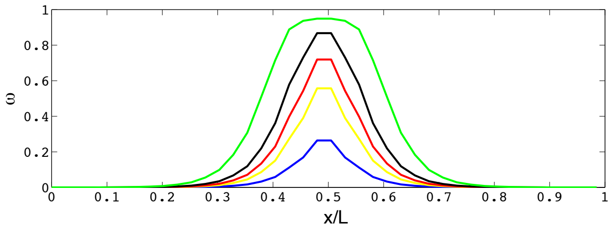

Fig. 2: Evolution of damage profile for formulation 2

approach relies on the plasticity formulated in the nominal (i.e. damaged) stress space. In the first case, the evolution of damage and the effective yield stress is prescribed, while in the second case the evolution of the nominal yield stress and damage is prescribed. Since the nominal stress is directly available from the stress-strain diagram, it may be simpler to describe it directly and then consider the effective yield stress as a derived quantity. The models are fully equivalent; however, it is neccesary to pay attention when constructing the nonlocal extension.

4. Implicit-gradient regularization

Here we focus on the regularization of the coupled damage-plastic models by the implicit-gradient formulation, with nonlocal cumulated plastic strain. In the regularized implicit-gradient formulation, the constitutive equations are enhanced by the nonlocal cumulated plastic strain, which is computed from a Helmholtz-type differential equation

$$\bar{\kappa} - l^2\nabla^2\bar{\kappa} = \kappa \tag{1}$$

where l is a length scale parameter and ∇ is the Laplace operator.

5. Numerical example

Simulation of a one-dimensional bar in tension is carried out to demonstrate regularization properties of different implicit-gradient formulations of plasticity coupled to isotropic damage. Influence of the nonlocal formulation on the profile of damage along the bar is studied. Isotropic linear hardening of the effective yield stress and exponetial evolution of damage is considered. At first, the over-nonlocal regularization based on nonlocal damage is considered, i.e. $\omega = g(\hat{\kappa})$. In this approach, the nonlocal cumulated plastic strain affects only the damage variable, while plasticity is formulated in the effective stress space and therefore remains local. The advantage of this approach is in a simple implementation based on the local return mapping algorithm followed by an explicit evaluation of the damage variable. The second class of models considered here is based on the over-nonlocal averaging of the nominal yield stress, $\sigma_Y = \sigma_Y(\hat{\kappa})$. Fig. 1 and Fig. 2 show the distribution of damage along the bar for different stages of loading for the first approach and the second approach, respectively.

6. Conclusions

We have presented two formulations coupling plasticity with damage, and introduced two different implicit-gradient regularization schemes which lead to an objective description of localized failure processes. We have shown that even if the local models are fully equivalent, the nonlocal formulation can lead to substantially different results; therefore it is neccesary to pay attention when constructing the nonlocal extension. Further research will focus on the comparison of the computational efficiency of both models, and on extensions of the gradient regularization to more general yield conditions.

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