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HIERARCHICAL MULTISCALE MODELLING OF POROUS MEDIA WITH APPLICATIONS IN BIOMECHANICS

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Abstract: *We consider materials with different levels of porosity at different scales. Homogenization theory provides a natural way of upscaling fluid-structure interaction problem posed at the smallest scale to higher levels of porosities in a sense that effective material coefficients (stiffness, permeability, Biot coefficients etc.) at a higher level are obtained by applying homogenization to the lower level. This approach leads to a convenient hierarchical description of the porous medium, suitable for multiscale modelling - in the contribution we present numerical examples motivated by bone tissue poromechanics.*

Keywords: ***poroelasticity, homogenization, multiscale modelling, double porosity.***

1. Introduction

Porous fluid saturated materials with different levels of porosities are abundant in nature and can be engineered as well to conform with requirements in technical practice. The paper describes one approach to modeling of the mechanical behavior of fluid-saturated cortical bone tissue. Namely, we provide homogenization-based formulae for computing the poroelasticity coefficients for a given geometry and topology of micro- and mesoscopic levels. A numerical example is presented illustrating the influence of the pore geometry on the two-level "micro-meso-macro" simulation results.

2. Hierarchical model of double porosity

In the paper, we describe an arrangement of two porosities (micro- or α-level and meso- or β-level) mutually separated by a semipermeable interface, however, each one forming a separate connected system. Then the homogenized problem results in two different pressures. At the mesoscopic scale we take into account the Darcy flow in the poroelastic matrix formed by α-level microstructure, although in the mesoscopic channels the fluid is assumed to be static with no pressure gradients.

Homogenization at each scale level (α and β), proceeds in two steps: 1) Find effective (homogenized) coefficients by solving auxiliary problems for several characteristic (or corrector) functions, cf. Rohan et al. (2012b); Rohan and Cimrman (2011); 2) Compute the homogenized coefficients that can be used for the higher level and/or "global" (homogenized) model of the current level. Due to linearity of the problems, those steps are decoupled in a sense that the computation of the homogenized coefficients for the global level is valid for any point having the corresponding "microstructure". Finally, the homogenized coefficients of the β level are used in the homogenized "macroscopic" problem.

3. Results

For numerical illustration of effects of connected porosities geometry we use the reference periodic cells shown in Fig. 1. The macroscopic problem solution (ramp-and-hold compression test) then leads to different time histories of micro-pore pressure p^α, meso-pore scalar pressure $\bar{p}^\beta$ and displacements $\boldsymbol{u}$, see Fig. 2, for the three cases of the α-level microstructure.

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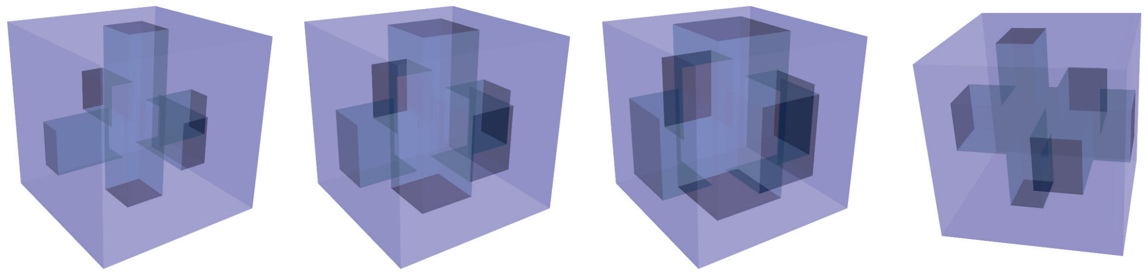
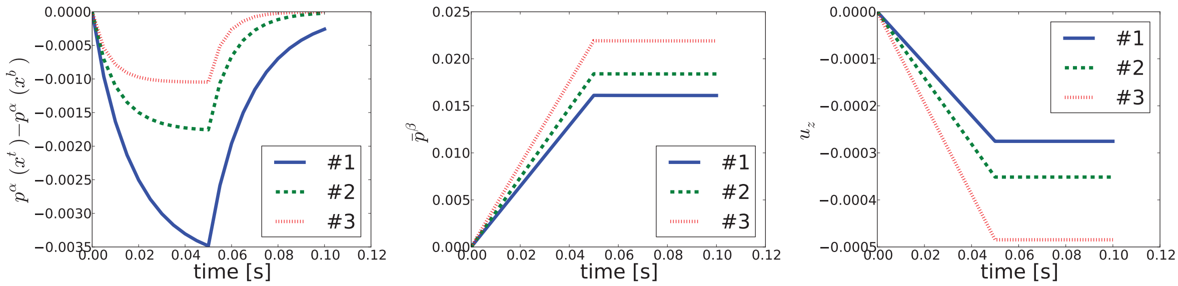


(a) (b) (c) (d)

Fig. 1: (a)-(c): reference cells for level α, case #1, #2 and #3. (d) reference cell with connected porosity for level β for all cases.



(a) (b) (c)

Fig. 2: Comparison of time histories of macroscopic solutions for the three cases: (a) difference between p^α in a point on the top face x^t and bottom face x^b; (b) $\bar{p}^\beta$, (c) z-displacement in x^t.

4. Conclusion

We have developed a two-level homogenized model of poroelastic media with weakly permeable interface between the two porosities, aimed to describe hierarchical structure of pores in the canaliculo–lacunar porosity of bone. The homogenization procedure makes possible to treat an arbitrary geometry and topology of the pores, whereby the localization tensors and coefficients can be calculated as the response of the autonomous microscopic problems; this was demonstrated using a numerical example computed by our code (Cimrman and contributors (2012)). The assumption of the weakly permeable interface disables full connection of the two porosities; this situation is treated in a separate paper Rohan et al. (2012a), cf. Rohan et al. (2012b) for related issues of the double porous materials.

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