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METHOD FOR CONSTRAINED DESIGNS OF EXPERIMENTS IN TWO DIMENSIONS

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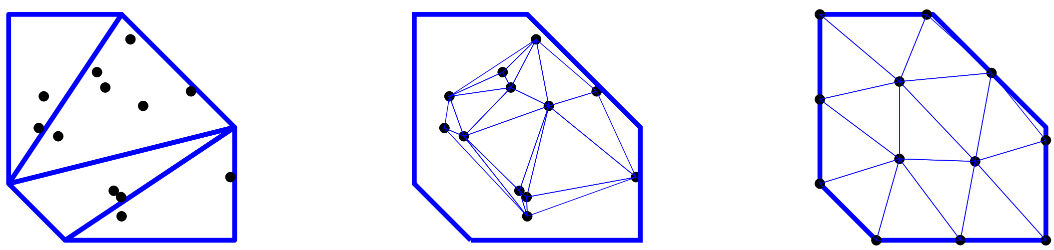
Abstract: *This paper presents a new approach for generating a Design of Experiments in constrained and non-regular two-dimensional spaces. The methodology is based on the triangulation of the admissible space by Delaunay Triangulation method. Then, a heuristic smoothing method for generating uniform Finite Element meshes within the triangulated space is applied to obtain uniformly spaced designs. Although not 100% reliable, the proposed method can produce superior designs to already known optimal solutions.*

Keywords: ***Design of Experiments, constrained design spaces, non-regular design spaces, space-filling, Delaunay triangulation.***

1. Introduction

Space-Filling Design Strategies known as *a Design of Experiments* (DoE) constitute an essential part of any experimentation. Our contribution is aimed at one particular domain of constrained design spaces. The most frequent example is the case of *a mixture experiment*, where individual inputs form a unity volume or unity weight (Montgomery, 2000, Chapter 11-5). This only condition leads to the simplex space; further limits of individual inputs then form *a polytope*, still convex but generally irregular space. Therefore, all traditional DoEs (Montgomery, 2000) that are constructed for hypercube spaces cannot be applied here.

Although the problem of mixture experiments is known for decades, the progress of methods for DoEs does not follow current developments. The main difference between classical and modern DoEs is the number of samples where, for the latter, the hundreds of samples is a usual scenario. Then, the classical approaches based on fixed small-sample templates (Montgomery, 2000) cannot be used. Up-to-date, the authors have found only few references on DoEs in constrained design spaces. Reference (Petelet et al., 2010) apply traditional Latin Hypercube (LH) designs to a bounding box followed by a Genetic



(a) Triangulation of the domain with randomly generated points.

(b) Triangulation of random points forming a truss-like structure.

(c) The final design after the application of the `Distmesh` tool.

Fig. 1: The generation of a uniform mesh from randomly generated points.

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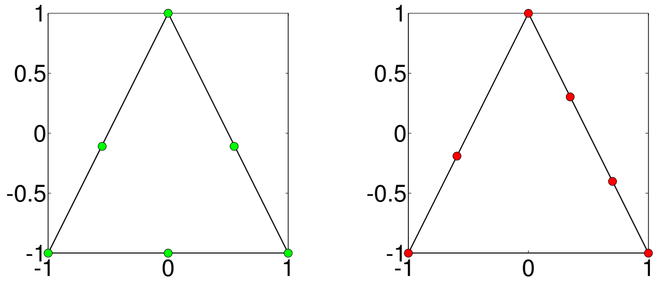

Fig. 2: Comparison of designs for Example 1: Reference design with Dopt = -50.0598 (Left), our best result Dopt = -53.0001 (Right). Lower value is better.

Algorithm (GA) to fulfil original constraints. Another approach is presented in (Hofwing and Strömberg, 2010), where interesting points are found by a GA and then, the final solution is located by sequential linear programming. The solution is general, however, the computational demands are enormous.

2. Method using Delaunay triangulation (DT) and `Distmesh` tool (DM)

In this contribution a different approach based on *Delaunay triangulation* (DT) of an admissible domain and an utilization of nice properties of the `Distmesh` tool (DM) (Persson and Strang, 2004) is presented, see Fig. 1. In our method the domain described by corner vertices is triangulated by DT and the desired number of random points is generated inside, see Fig. 1a). Then the DM tool is applied. The `Distmesh` tool is a heuristic smoothing algorithm for generating uniform meshes that is based on a simple dynamical system of expanding pin-jointed structure, here characterized by the second mesh, see Fig. 1b). Those trusses that are too short are causing repulsive forces that move the too close nodes apart, see Fig. 1c) for the final solution.

3. Conclusions

Our results are compared to seven constrained examples in two dimensions presented in (Hofwing and Strömberg, 2010), namely a placing of six design points in a triangle, parallelogram, pentagon, hexagon, heptagon, octagon and a search for a position of twelve design points in an irregular hexagon. Although not designed directly for space-filling optimization, our procedure is able to outperform the reference algorithm even from the D-optimal point of view, see Fig. 2.

References

Hofwing, M. and Strömberg, N. (2010). D-optimality of non-regular design spaces by using a Bayesian modification and a hybrid method. *Structural and Multidisciplinary Optimization*, 42:73–88.

Montgomery, D. C. (2000). *Design and Analysis of Experiments, 5th Edition*. Wiley, 5 edition.

Persson, P.-O. and Strang, G. (2004). A simple mesh generator in MATLAB. *SIAM Review*, 46(2):329–345.

Petelet, M., Iooss, B., Asserin, O., and Loredo, A. (2010). Latin hypercube sampling with inequality constraints. *AStA Advances in Statistical Analysis*, 94:325–339.