Abstract: The occurrence of shrinkage in concrete leads to development of internal tension stresses which can result in concrete cracking. The presence of cracks in concrete creates pathways that ease the access of aggressive agents reducing concrete structure durability and service life. Consequently, the correct reduction of shrinkage strain during the designing process is important to assure the structure’s durability and long time serviceability. In light of this, the objective of this research was to develop an experimental based fuzzy logic model to predicting self-compacting concrete shrinkage. The fuzzy logic model decision-making is optimized through an evolutionary computing method, therefore enhancing computational effectiveness. The obtained results are compared to the B3 shrinkage prediction model and statistical analysis, indicating the reliability of the proposed model, are presented.

Keywords: Fuzzy logics, shrinkage, self-compacting concrete, evolutionary computing

1. Introduction

Concrete shrinkage is defined as decrease in concrete volume with time. This volume decrease does not depend on external stress and it is not completely reversible. The shrinkage in concrete is associated with a series of factors, such as chemical reaction, gradient in temperature, and movement and loss of water. Each one of these factors leads to different types of shrinkage, such as, autogenous, plastic, drying and thermal shrinkage, among others. For comprehensive review see Brooks, J., 2003, and Mehta & Monteiro, 2006, publications.

The occurrence of shrinkage leads to development of internal tension stresses, which may result in concrete cracking. The presence of cracks in concrete creates pathways that ease the access of aggressive agents into concrete and contributing to reduction in concrete structure durability. An example of concrete structure damaged by the occurrence of excessive shrinkage is illustrated in Fig. 1.

![Fig.1: Foundation block of a residential building in Prague, Czech Republic: (a) construction side overall view and (b) detail of 0.8mm crack width caused by concrete shrinkage.](a) (b)

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Hence, it can be noticed that a trustworthy definition of concrete shrinkage strain is important in the designing process of structures, since it helps reducing maintenance costs and ensures that the specifications of expected service life and durability requirements will be fulfilled.

Self-compacting concrete, SCC, is a high-performance concrete that can flow under its own weight so as to completely fill the formwork and self-consolidate without any mechanical vibration (Erdem et al., 2009, Gaimster et al., 2003). SCC is specifically designed to achieve excellent deformability, a low risk of blockage, and good stability, ensuring a high formwork filling capacity. The use of SCC is rapidly developing in the construction industry, most likely due to the production process costs and the advantageous organizational nature of this material.

Nonetheless, it is important to consider that the production of SCC is more difficult than that of conventional concrete and many parameters have to be considered in order to obtain a final product that has an acceptable quality for the intended purpose. When compared to conventional concrete, SCC mixtures require a higher volume of cement paste in the composition to achieve excellent deformability and high formwork filling capacity. Then, considering that shrinkage is a result of hydration reaction in the cement paste, SCCs are likely to present higher values of shrinkage strain. Hence, measuring shrinkage strain in SCC mixtures arises as a relevant issue.

Usually, experimental measurements are required to determine the concrete shrinkage strain. However, measurement of shrinkage strain is laborious, time consuming and expensive, therefore construction designers tend to use shrinkage prediction models. These models aim to determine concrete shrinkage strain in a faster and less expensive way when compared to experimental measurements. Amongst several existing prediction models, the B3 model has been selected for comparison in this research. The B3 model was developed in 1995 by Bažant (Bažant, 1995), this model has been updated over time and its latest version dates back to 2000, (Bažant et al., 2000).

Though regularly used, shrinkage strains obtained from the prediction models do not necessarily match experimental measurements. The comparison of experimentally measured and predicted, shrinkage strain curves by the B3 model are presented in Fig. 2. The experimental curves, shown in Fig. 2a and b, consist of a part of experimental data from published by Al-Attar, 2008 and Brook et al., 2001, for different types of concrete.

![Comparison of predicted and experimental shrinkage strain curves](image)

**Fig. 2: Comparison of predicted and experimental shrinkage strain curves for (a) Conventional Concrete – input data from Al-Attar, 2008, and (b) High Strength Concrete – input data from Brooks et al., 2001.**

From Fig. 2, a considerable difference from experimental and predicted shrinkage strain can be noticed, showing the relative error of B3 model in predicting shrinkage strains. This way, it can be stated that the reliability of this prediction model is open to discussion, and improvements are required.

In light of this, the present work aims to develop a methodology for defining an experimental-based prediction model for SCC shrinkage. In this study, Soft-computing techniques, particularly fuzzy logic systems and evolutionary computing, were used to develop a modelling methodology, which was then applied to build a SCC shrinkage strain prediction model. Further, through statistical analysis, the results from the obtained model were compared to other published data.
2. Fuzzy logic and evolutionary computing

Fuzzy theory, first introduced by Zadeh in 1967, (Zadeh, 1967), correspond to a natural way of thinking where verbally expressed rules are applied to deal with vagueness, imprecision and ill-defined data. Basically, fuzzy logic control systems comprise three steps: fuzzification, decision-making and defuzzification. The fuzzification consists of converting the crisp input values into degrees of membership by means of input membership functions. This step is followed by the decision-making, where a degree of membership is assigned to the output variable based on the rule based and output fuzzy sets. Finally, the defuzzification is processed to convert the output fuzzy set in to a single value, in this case, the predicted shrinkage strain value.

The key factors to achieve an acceptable performance in a fuzzy logic system are connected to the definition of the number of fuzzy sets and the shape of the membership functions. Commonly, there are \( x^y \) fuzzy rules, where \( x \) and \( y \) are the number of sets and input variables, accordingly. In the classical fuzzy logic approach, the number of fuzzy rules can be reduced by the user’s experience, and the shape of membership functions is usually adopted as linear to simplify calculations. This approach application has been used by Štemberk et al., 2011 to simulate heat evolution during hydration of typical Portland cement.

Nevertheless, when the classical approach is implemented to model non-linear materials behavior, the final results are a rather rough shaped piecewise curve as indicated in Fig.3. Note that, the use of the classical fuzzy logic approach is also feasible to model non-linear materials behavior. However, a considerable larger number of linear fuzzy sets is required to obtain smoothed curves, thus leading to a longer data collection time and high computational cost. In order to improve the entire modelling process a modified approach, which includes evolutionary computing methods, is proposed in this research.

Evolutionary computing comprises robust optimization methods that can be generally applied without recourse to domain-specific heuristics. These methods operate on a population of potential solutions and apply the principle of survival of the fittest to produce successively better approximations to a solution, (Coley, 1999). Amongst several Evolutionary computing methods, Genetic Algorithms, \( GA \), have been successfully applied for numerical optimization in civil engineering, e.g., Rokonuzzaman et al., 2010 presents an application of \( GA \) for calibration of parameters for a hardening-softening constitutive model.
**GA**s consist of adaptive heuristic search algorithms based on the principles of Darwin’s theory of natural selection. They represent an intelligent exploitation of a random search that uses historical information to guide the search into the region of better performance, within a defined search space. The basic form of a **GA** involves three operators to achieve evolution: selection, or reproduction, crossover and mutation, (Coley, 1999).

3. **Proposed methodology for optimization of fuzzy decision-making**

The proposed methodology combines fuzzy logics and genetic algorithms to optimize fuzzy decision making, which is achieved by optimizing the shape of the membership functions. Bearing in mind this idea, and focusing on SCC shrinkage, the following methodology is proposed:

Initially, the user has to define the number of representative intervals, $N_{int}$, of shrinkage strain, $e_{sh}$, and concrete age, $t$, for experimental shrinkage strain curves obtained from concrete mixtures with different volumes of cement paste. Note that, the more complex the shape of the curve the higher the number of intervals necessary to achieve optimal results. In the sequence, the user specifies the size of the population, $S_{pop}$, defined as 10, which will be used in the genetic part of the algorithm. The optimization process is from this point on an automatic process. Based on the value set for $N_{int}$, the encoding of each individual, or chromosome, from the population is defined. The encoding comprises a string of $n_{enc} = 2 \times N_{int}$ real numbers, which correspond to the exponent values, $E_L$ and $E_R$, from each membership function to be optimized, see Fig.4.

![Fig. 4: General equation and shape of the membership functions to be optimized.](image)

Subsequently, an initial random population is generated and the fitness function, $f(x)$, is evaluated. The fitness function corresponds to the $MSE$ function described in Eq. (1).

$$f_j = \sqrt{\frac{1}{n-1} \cdot \sum_{j=1}^{n} d_j^2} ,$$

where $n$ is the number of data points considered in the analysis, $d_j$ is the percent difference between each predicted and measured data point, $f_j$ is the mean square error for data set $j$, and $f_{all}$ is the overall mean square error, computed by means of Eq.(2).

$$f_{all} = \sqrt{\frac{1}{N} \cdot \sum_{j=1}^{n} f_j^2} ,$$

with $N$ as the total number of data sets.

Next, three genetic operators: selection, crossover and mutation, are applied to generate a new population. The selection operator chooses the chromosomes in the population for reproduction. In this case, the tournament selection scheme, which selects the best fitness from individuals chosen at random from the population, was applied. The selected chromosomes, or parents, are then crossed...
over by one-point crossover scheme, with a probability, $C_{\text{prob}}$, set as 90%, to create a new individual to be included in the population. This scheme sets an independent randomized crossover point for couples of parents, whose data is swapped to create a new population. After that, a mutation operator is applied to maintain genetic diversity. The mutation is performed by disturbance with a probability, $M_{\text{prob}}$, set as 10%. The mutation operator randomly flips some of the values in a chromosome to create a mutated version of the individual to be incorporated in the population. After a new population has been generated, the fitness function re-evaluates all individuals from the new population. The obtained results, $f(x')$, are then compared with those from previous populations. Further, elitism is applied, i.e., the best overall solution is stored. In case none of individual from the new population shows better fitness than the stored solution, the individual with the worst solution from the new population is replaced by the best overall solution. The automatic process of generating a new population and evaluating the best fit is repeated until convergence occurs. Convergence was considered as achieved when more than 200 consecutive runs do not lead to any improvements in the fitness function result. The final result consists of a group of optimized fuzzy sets which will compose the fuzzy decision-making. Once the group of optimized fuzzy sets is defined, the decision making is then based on the rule base, $R^m$, defined in (3), and the final predicted shrinkage strain, $e_{\text{sh, output}}$, is computed by means of Eq.(4).

$$ R^m : IF V_{cp} \text{ is } V_{cp,nr} \text{ THEN } e_{\text{sh}} \text{ is } e_{\text{sh,nr}}, $$

$$ e_{\text{sh, output}} = \frac{\sum_{nr=1}^{nr} \mu_n \cdot e_{\text{sh,nr}}}{\sum_{nr=1}^{nr} \mu_n}, $$

where $nr$ is the number of rules; $V_{cp}$ is the cement paste volume input value, in $l/m^3$; $V_{cp,nr}$ and $e_{\text{sh,nr}}$ are the optimized group of fuzzy sets for cement paste volume and shrinkage strain, respectively; $\mu_{nr}$ is the degree of membership assigned to the group $e_{\text{sh,nr}}$ from each rule $R^m$.

The methodology described in this section was applied for the experimental data presented by Leemann et al., 2011, and the results from Loser et al., 2009, were used to verify the optimized model.

4. Results and discussion

In the present analysis the volume of cement paste was chosen as input parameter and two experimental curves, illustrated in Fig.5, were considered as training data. The curves from Fig.5 were taken from the experimental database presented by Leemann et al., 2011.

![Fig. 5: Shrinkage strain curves of SCC mixtures from Leemann et al., 2011.](image)

The proposed methodology was performed for this data and convergence was achieved after approximately 500 iterations. The fuzzy logic prediction model for SCC, named FL-I model, is then composed by the optimized fuzzy sets, the rule base, and the final output equation, presented in Eq.(4). The graphical representation of the FL-I model, illustrated in Fig.6, indicates the exponent values of the membership functions and the rule base use by the model to predict the SCC shrinkage strain curve.
Fig. 6: Graphical representation of FL-I model.
Since only two curves were available for the optimization process, the fuzzy sets connected to the volume of cement paste had to be set as a linear functions, see Fig.6. It is also important to observe that the obtained model is suitable for predicting shrinkage strain up to 90 days and testing conditions defined by Leemann et al., 2011.

The experimental data published by Loser et al., 2009 was used to verify the quality of the FL-I model in predicting shrinkage strain. This data comprises shrinkage curves of five different SCCs with testing conditions compatible to the limits defined to the FL-I model. The volume of cement paste of each SCC, necessary as an input parameter, is listed in Table 1. Moreover, the shrinkage strain curves of each SCC mixture from Loser et al., 2009 were also compared to the strain curves obtained from the B3 prediction model. For that, the input data shown in Table 1 was used.

Table 1. Input data used to predict shrinkage strain based on B3 and FL-I model, (Loser et al., 2009).

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>SCC</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>V</td>
</tr>
<tr>
<td>Cement paste volume, $V_{cp}$ [l/m$^3$]$^*$</td>
<td>329.0</td>
<td>349.0</td>
<td>316.0</td>
<td>342.0</td>
<td>332.0</td>
</tr>
<tr>
<td>Design compressive strength, $f_{c'}$ [MPa]</td>
<td>53.3</td>
<td>63.1</td>
<td>51.0</td>
<td>49.4</td>
<td>66.0</td>
</tr>
<tr>
<td>Compressive strength at 28 days, $f_{cm28}$ [MPa]</td>
<td>61.3</td>
<td>71.1</td>
<td>59.0</td>
<td>57.4</td>
<td>74.0</td>
</tr>
<tr>
<td>Curing time, $t_c$ [days]</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Relative Humidity, RH [%]</td>
<td>70.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cement type</td>
<td>CEM I 42.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specimen size [mm]</td>
<td>$120 \times 120 \times 360$ (Shape: infinite prism)</td>
<td></td>
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</tbody>
</table>

* Considered as an input parameter of the FL-I model;

The $MSE$ values, computed by Eq.(1), for the FL-I and the B3 model are presented in Table 2.

Table 2. Individual and overall $MSE$ values for different shrinkage prediction models.

<table>
<thead>
<tr>
<th>$f_{model}$ [%]</th>
<th>SCC</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>V</td>
</tr>
<tr>
<td>$B3$</td>
<td>31.3</td>
<td>27.4</td>
<td>29.0</td>
<td>17.2</td>
<td>32.9</td>
</tr>
<tr>
<td>$FL-I$</td>
<td>6.9</td>
<td>19.1</td>
<td>4.4</td>
<td>16.2</td>
<td>14.1</td>
</tr>
</tbody>
</table>

From Table 2, it can be seen that the FL-I model presented the lower $MSE$ values than the B3 model in all cases. By using the values from Table 2 in Eq.2 an overall $MSE$ of 13.5% is obtained for the FL-I model, against 28.1% for the B3. Therefore, it can be concluded that, once the limits FL-I model are respected, the results predicted by the FL-I model are more reliable than B3.

Nonetheless, though the overall $MSE$ presented by the FL-I model was lower than the B3, its value is still considered high, around 15%. The reason for that is probably because only two experimental curves were used as training data. If an intermediary curve was included in the training data set, the linear shape of the $V_{cp}$ fuzzy set, see Fig.6, would be optimized. Consequently, the final $MSE$ of FL-I model would be even lower than observed.

To verify this assumption, one of the experimental curves from Loser et al., 2009, more specifically SCC IV, see Table 1, was included in the training data. The optimization process was again performed and the exponent values, $E_R$ and $E_L$, obtained for the optimized fuzzy sets are indicated in Fig.7. This leads to a new prediction model, called FL-II model. It is important to observe that the experimental data from SCC IV was only used as training data to optimize the shape of the fuzzy set of cement paste volume, therefore the optimized fuzzy sets connected to the shrinkage strain, $e_{sh}$, and the rule base, presented Fig.6, remained the same.

Once more, the experimental and predicted shrinkage strains, from FL-II model, were compared and the $MSE$ values were computed by Eq.(1). The obtained results are presented in Table 3 together with the $MSE$ values from FL-I model.
Table 3. Individual and overall MSE values for different shrinkage prediction models.

<table>
<thead>
<tr>
<th>$f_{model}$ [%]</th>
<th>SCC</th>
<th>$f_{overall}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>$FL-I$</td>
<td>6.9</td>
<td>19.1</td>
</tr>
<tr>
<td>$FL-II$</td>
<td>8.9</td>
<td>3.8</td>
</tr>
</tbody>
</table>

From Table 3, it can be seen that the overall MSE value for the $FL-II$ are considerably lower than the $FL-I$ model. Hence, the assumption that including additional training data would lead to a prediction model with lower error is verified. Although higher than the $FL-I$ model in one case, the individual values of MSE for the $FL-II$ were always below those obtained for the $B3$ model.

Finally, the lower MSE values from $FL-I$ and $FL-II$ models, compared to the $B3$ model, confirm their quality to simulate materials behaviour, and also the success in combining fuzzy logics and genetic algorithms to build optimized materials models. The obtained model are suitable to predict SCC shrinkage strain within the limits of the model, excluding the need of additional experimental analyses.

5. Final considerations

By defining a shrinkage strain prediction model for SCC the objective of this paper has been achieved. Based on the presented results the following conclusion can be drawn.

The use of the proposed methodology for optimization of fuzzy decision-making has shown satisfactory results. The optimized group of fuzzy sets led to a proper prediction of the shrinkage curves with a reduced number of rules, making the modelling process more effective.

The statistical analysis leads to overall mean square error around 30% for the $B3$ model, against 13.4% for the $FL-I$ model, indicating that the $FL-I$ model better represents the materials behaviour and can be used to predict SCC shrinkage once the limits of the model are respected. The further inclusion of additional training data in the optimization methodology contributed to reduce the overall error of the model from ~15% to ~7%, demonstrating the flexibility of the model in self-adjusting according to the training data. Such flexibility is a great advantage of fuzzy logic-based model when compared to the prediction models that are based on equations and its constants.

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