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SEQUENTIAL DESIGNS OF EXPERIMENTS FOR SAMPLING-BASED SENSITIVITY ANALYSIS

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Abstract: *Widely used strategy to assess the sensitivity is based on a set of simulations for a given sets of input parameters, i.e. points in the design space. The accuracy of the sensitivity prediction depends on the choice and the number of design points called the design of experiments. Moreover, once the design of experiments is created, the obtained sensitivity prediction may be inaccurate because of the insufficient number of design points. To improve the prediction, new design points should be sequentially added into the existing design.*

Keywords: ***Sequential design of experiments, Space-filling, Orthogonality, Latin Hypercube Sampling, Sampling-based sensitivity analysis***

1. Introduction

Sensitivity analysis (SA) is an important tool for investigating properties of complex systems. To be more specific, SA provides some information about the contributions of individual system parameters/model inputs to the system response/model outputs. The presented contribution is focused on widely used sampling-based approaches (Helton et al (2006)), particularly aimed at an evaluation of Spearman's rank correlation coefficient (SRCC), which is able to reveal a nonlinear monotonic relationship between the inputs and the corresponding outputs.

When computing the SA in a case of some real system using expensive experimental measurements or some computationally exhaustive numerical model, the number of samples to be performed within some reasonable time is rather limited. Randomly chosen sets of input parameters do not ensure appropriate estimation of related sensitivities. Therefore the sets must be chosen carefully. A review and comparison of several criteria, which can govern the stratified generation of input sets called as a design of experiments (DOE), is presented in Janouchová and Kučerová (2011).

Another important aspect of a DOE generation is a choice of the number of design points. A small DOE does not have to give us the required accuracy of the sensitivity prediction and one has to increase the number of design points so as to achieve the accuracy improvement. Once having the time-consuming measurements for the original small design, adding new points into the existing design is more efficient than generation of the whole larger DOE. In this paper we follow the results presented in Janouchová and Kučerová (2011) for small DOEs and focus on a humble goal to compare the qualities of sequential designs obtained by sequential addition of a constant number of new points, preserving the original discretisation and optimized to the individual criteria.

2. Results

There are two methods presented for generating sequential designs in this paper. The first one is based on unrestricted selection of new points according to the optimized criterion and both the free DOEs as well as LH DOEs were employed as initial ones. The second method preserve the LH constrains also for the added points and thus, an equal number of points are located in each column or row. Here the LH designs are used as the initial designs.

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In engineering practice, the majority of the numerical models fulfil the condition of a monotonic relationship between the model parameters and the model response. Therefore, to support the study of optimal DOE quality in sampling-based SA, we performed a comparison for a list of nonlinear but monotonic models.

Then, the parameter-response correlations were estimated using the all obtained optimal designs and the differences among correlations obtained by the optimal designs and correlations obtained by the full designs are stored. The error measure in the parameter-response correlations evaluated for a given function is considered as an average difference between each parameter and model response correlation obtained by an optimal and a full design.

Tab. 1: The mean and maximal errors over all mathematical functions and designs with 10 up to 40 points multiplied by 100.

Method		**AE**		**EMM**		**$\mathbf{ML_2}$**		**Dopt**		**PMCC**		**SRCC**		**KRCC**		**CN**	
		mean	max	mean	max	mean	max	mean	max	mean	max	mean	max	mean	max	mean	max
1^{st}	free	3.2	17.6	4.8	20.6	3.0	13.5	3.2	21.2	4.9	32.5	4.4	31.8	4.4	30.5	4.5	34.1
	LH	3.4	18.1	6.1	22.5	2.8	10.6	3.9	17.4	3.8	23.1	3.7	25.1	3.7	23.4	3.5	24.2
2^{nd}	LH	4.2	16.2	6.8	24.6	2.8	9.0	4.4	14.5	3.1	21.3	3.2	19.4	3.2	21.2	3.1	22.9

The next sensitivity analysis study is devoted to illustrative engineering problems with higher number of dimensions. We have chosen two models of truss structures commonly used as benchmarks for sizing optimization.The first one represents a ten-bar truss structure (Venkaya (1971)) and the second model concerns a 25-bar truss structure. The response of these models consists of three components: total weight of the structure w, maximal deflection d and maximal stress s. Because of higher dimensions of these problems, we have generated the optimal DOEs only with LH restriction and these results are summarized in Table 2.

Tab. 2: Mean and maximal errors in correlation predictions for structural models.

Model	**Points**	**AE**		**EMM**		**$\mathbf{ML_2}$**		**Dopt**		**PMCC**		**SRCC**		**KRCC**		**CN**	
		mean	max	mean	max	mean	max	mean	max	mean	max	mean	max	mean	max	mean	max
10-bar	42	4.9	7.5	4.7	7.7	4.6	13.9	4.7	8.0	5.7	10.3	5.7	11.0	5.7	10.7	21.5	29.7
	84	3.3	5.7	3.9	6.7	2.7	7.7	3.2	5.8	3.9	6.7	3.6	7.7	3.8	8.0	4.5	8.0
25-bar	30	25.7	32.3	26.2	33.5	23.9	28.9	27.9	38.3	25.9	30.8	25.6	30.7	25.6	31.4	26.0	31.2
	60	25.2	30.9	25.7	31.9	23.2	29.5	25.9	34.1	25.1	30.4	25.0	30.4	25.5	31.4	25.3	30.9

3. Conclusions

This paper compares the sequential designs optimized according to one of the eight criteria and inspects their suitability for application in a sampling-based SA. The presented results revealed that the ML_2 criterion can give very good results, this criterion is very robust and thus the obtained DOEs provided very small errors in sensitivity predictions with very small variance. The LH designs optimized with respect to ML_2 criterion provide better results. Therefore they can be recommended for the practical usage.

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