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AN ENHANCED NUMERICAL SOLUTION OF BLASIUS EQUATION BY MEANS OF THE METHOD OF DIFFERENTIAL QUADRATURE

A. Guran*, J. Gwinner**

Abstract: *The differential quadrature method (DQM) is used to solve the two-dimensional Blasius boundary layer problem which is described by a third-order nonlinear differential equation. The governing nonlinear equation of boundary-value Blasius problem is first converted to a pair of nonlinear initial-value problems and then solved by both the DQ method and classical fourth-order Runge-Kutta method (RK4). It is revealed that as compared to the RK4, the DQ method can achieve much higher order of accuracy for the numerical results using larger time step sizes.*

Keywords: ***Differential quadrature method (DQM), Blasius equation, Fourth-order Runge-Kutta method (RK4), Convergence and accuracy of DQ solutions.***

The Blasius boundary layer is an example of two-dimensional boundary layer problems. The Blasius problem models the behavior of two-dimensional steady state laminar viscous flow of an incompressible fluid over a semi-infinite flat plate. The governing equation of the problem is

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0 \quad , \ 0 \le \eta \le \infty \tag{1}$$

where η and f(η) are the dimensionless coordinate and stream function, respectively (Schiliching, 2004). The boundary conditions for equation (1) are

$$f(0) = f'(0) = 0 \quad , \quad f'(\infty) = 0 \tag{2}$$

The problem was first solved by Blasius using a series expansions method. But the proposed analytic series solution does not converge at all. In fact, the obtained analytic solution is valid only for small values of η (i.e., the series solution converges only within a finite interval [0, η0] where η0 is an unknown constant which can be determined numerically or analytically). Howarth (1938) solved the Blasius equation numerically and found η0 ≈ 1.8894/0.33206. Furthermore, Asaithambi (2005) solved the Blasius equation more accurately and obtained this number as η0 ≈ 1.8894/0.332057336. Due to the limitation of Blasius power series solution, many attempts have been made to obtain the solutions which are valid on the whole domain of the problem. Some researchers have solved the problem numerically and some analytically or semi-analytically. Applying the homotopy analysis method (Liao, 1997), Liao obtained an analytic solution for the Blasius equation which is valid in the whole region of the problem (Liao, 1998; Liao,1999). Using the variational iteration method (He,1997), He constructed a five-term approximate-analytic solution for the Blasius equation which is also valid for large values of η (He,1999). However, the solutions obtained were not very accurate. The Adomian decomposition method (ADM) has also been used by some researchers to find analytic solutions for the Blasius equation (Allan and Syam, 2005; Wang, 2004; Abbasbandy, 2007). A homotopy perturbation solution to this problem was presented by He (He, 2003; He, 2004). Fang et al. (Fang et al., 2006), Cortell (Cortell, 2005), Ahmad (Ahmad, 2007), and Ahmad and Al-Barakati (Ahmad, 2009) also solved the Blasius problem using various numerical and analytical methods. From the review of the proposed schemes in (Liao, 1997; Ahmad and Al-Barakati, 2009), two general limitations may be observed: (1) The proposed approximate-analytic methods can not yield accurate solutions when a rather small number of solution terms are used, (2) Many calculations should be done to construct the resulting semi-analytic solutions and this increases considerably the CPU time especially when a large number of terms of solutions are to be used. The above-mentioned limitations can be

* Professor, Technical University of Munich (Germany), Director, Institute of Structronic, 275 Slater Street, 9th Floor,Ottawa, Canada. Corresponding Author, ardeshir.guran@mcgill.ca

eliminated by using the higher-order methods such as the differential quadrature method (DQM).The DQ method is capable of yielding high accurate numerical solutions using very few grid points. So far, the DQ method has been efficiently employed in a variety of problems in engineering, mathematics, and physical sciences and is emerging as a powerful numerical discretization tool. However, the DQ method has its difficulty in implementation to the problems with irregular and complex geometry. Since the time domain has no such difficulty, the strengths of higher-order accuracy of the DQ method can be fully exploited in approximating the time derivatives.

In this work, we first convert the Blasius equation to a pair of initial-value problems (Na, 1979) and then solve the pair of initial-value problems by the DQ method. The resultant initial-value problems are also solved by the conventional fourth-order Runge-Kutta method (RK4). The efficiency, accuracy, and convergence of the DQ time integration method are then investigated and analyzed. It is demonstrated that the DQ method gives better accuracy than the RK4 using much larger time steps.

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