21st International Conference **ENGINEERING MECHANICS 2015**
Svratka, Czech Republic, May 11 –14, 2015
Paper **#133, pp. 220–221**

Parallel Solver for Large Thermo-Elasto-Plastic Finite Element Problems

Petr Pařík

Institute of Thermomechanics, AS CR, Dolejškova 1402/5, 182 00 Prague 8, Czech Republic

parik@it.cas.cz

Keywords: sparse direct solver, finite element method

Abstract: The article deals with the implementation and verification of a numerical solver for large heat transfer problems in the context of the finite element analysis. Large problems are defined as problems whose solution on contemporary computers is computationally difficult. The solver builds on the numerical methods for an efficient direct solution of large linear elastostatic problems, developed in earlier works. The solver is integrated within our in-house finite element code PMD. Verification is carried out on several large thermo-elasto-plastic problems from real-world engineering practice.

Introduction

Direct solution methods [1] are particularly important in the Finite Element Method (FEM) [2] since they enable the factorization (the most time demanding part of the solution) to be performed only once for a given problem and subsequently to solve it for any number of right-hand sides (load cases), which is especially useful in complex problems (thermo-elasto-plastic, dynamic, etc.).

The presented approach to an efficient implementation of a sparse direct solver focuses on minimizing the required amount of both the storage space and the computational time required for the direct solution of a large problem. This is achieved particularly by using a modified minimum ordering algorithm to reduce the fill-in, therefore, reducing the number of numerical operations needed to compute the solution [3]. Both the factorization and the substitution algorithms are parallelized to fully exploit today's multi-core and multi-processor computers. The global coefficient matrix is assembled and stored in-core throughout the computation, using an efficient sparse matrix storage format. Although that leads to considerably high memory requirements, in practice, it does not present a significant problem for today's computers. The finite element Fortran code PMD [4] is used as the framework for the presented implementation of a sparse direct solver.

Problem formulation

The solution of a linear equation system can be written in the matrix form simply as

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

where $\mathbf{A}$ is the coefficient matrix, $\mathbf{x}$ is the unknown solution vector and $\mathbf{b}$ is the righ-hand side vector. In solid continuum mechanics, $\mathbf{A}$ may refer to the stiffness or conductivity matrix, $\mathbf{x}$ to the displacement or temperature vector, and $\mathbf{b}$ to the external loading forces or thermal loading.

Moreover, especially in the context of FEM, a single problem almost always needs to be solved for many right-hand sides (load cases), thus, vectors $\mathbf{x}$ and $\mathbf{b}$ in Eq. 1 actually become matrices. Furthermore, the global coefficient matrix $\mathbf{A}$ obtained from the finite element discretization is singular, therefore, equation system Eq. 1 cannot be solved unless boundary conditions, which are a part of the finite element problem formulation, are taken into account.

For a complete solution of a combined thermo-elasto-plastic problem, iterative non-linear numerical solution methods are needed [2]. From the computational point of view, they involve matrix-vector and vector-vector operations that are however not as computationally intensive as the underlying solution of linear equation system, which plays major part in each iteration.

Results

In Table 1, selected largest computed problems are listed along with their order n (number of unknowns), block-order N (number of diagonal blocks), frontwidth n_{fw}, number of nonzero matrix blocks N_{nz} and number of nonzero matrix elements n_{nz}.

Computational times are presented in Table 2, where the sparse direct solver clearly demonstrates its efficiency over the existing frontal solver.

Table 1: Test problems‘ parameters.

Problem no.	n	N	n_{fw}	N_{nz}	n_{nz}
1	1,129,747	257,861	7,582	13,548,221	216,454,160
2	1,739,211	579,737	4,719	124,138,819	1,115,504,693
3	1,909,577	439,287	5,214	29,584,204	484,886,739
4	1,973,550	657,85	12,873	218,994,791	1,968,939,519
5	2,858,631	952,877	18,828	667,395,241	6,003,699,588
6	3,022,848	695,05	11,342	65,174,392	1,063,250,512

Table 2: Solution time for test problems.

Problem no.	Frontal solver [hrs]	Sparse direct solver [min]
1	4.49	17
2	3.56	49
3	4.12	47
4	22.38	115
5	> 99.99	253
6	17.17	120

Conclusion

A distinct approach to the direct solution of large thermo-elasto-plastic problems was presented. Efficient sparse matrix storage and necessary algorithms were implemented into a sparse direct solver, which was thoroughly tested on real-world engineering problems from solid continuum mechanics. The results obtained from the numerical tests confirmed the solver's efficiency and scalability. High memory requirements of the solver were anticipated and will present gradually less a problem with the ever-increasing capacity of computers.

The potential of the implemented sparse direct solver is fully utilized within the solution procedures for nonlinear heat transfer and inelastic finite element problems.

Acknowledgement: The work was supported by TA CR project No. TH01010772 within the institutional support RVO 61388998.

References

[1] A. George, J.W.H. Liu, Computer solution of large sparse positive definite systems, Prentice-Hall, Englewood Cliffs, NJ, 1981.

[2] K.J. Bathe, Finite Element Procedures, Prentice-Hall, Upper Saddle River, NJ, 1996.

[3] P. Pařík, An out-of core sparse direct solver for large finite element problems, CTU reports Vol. 1 No. 15, Czech Technical University Press, Prague, 2011.

[4] PMD version 11.0, information on http://www.pmd-fem.com/