18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 64–65
Paper #6

RESONANCE BEHAVIOUR OF SPHERICAL PENDULUM – INFLUENCE OF DAMPING

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Abstract: *Experimental and numerical model of a uni-directionally driven pendulum-based tuned mass damper is presented in the paper. Stability of the motion in a vertical plane is analysed in the theoretically predicted resonance region. For the experimental part, special experimental frame is used, allowing independent change of linear viscous damping in the both perpendicular directions. Mathematical model respects the non-linear character of the pendulum and allows to introduce asymmetrical damping. Sensitivity of the resonance behaviour on the change of damping in both directions is studied and commented in the paper. The stability of the system is analysed experimentally and compared with numerical and theoretical results.*

Keywords: ***spherical pendulum, tuned mass damper, damping***

1. Introduction

A typical tuned mass damper (TMD) has a form of a pendulum. This low cost passive device used at tall masts and towers is very popular for its reliability and simple maintenance. However, conventional planar linear model of such TMD is satisfactory only if the amplitude of kinematic excitation at the suspension point is very small and if its frequency remains outside a resonance frequency domain, which is possible only at the cost of lower efficiency of the damper. To improve the design of pendulum, a spherical pendulum should be considered.

The present article exploits the analytical approach to the subject described in (Náprstek and Fischer, 2009) and compares it with some experimental findings. The movement of the pendulum is described in two cartesian coordinates ξ, ζ, representing the projection of the pendulums bob to the x, y plane. The uni-directional harmonic excitation is supposed. A specially developed experimental rig is used to analyse a kinematically driven pendulum suspended from the Cardan joint. The damping can be arbitrarily adjusted by means of two independent magnetic units attached to the frame and to the supporting axes of rotation. The stability of the system is studied experimentally and numerically for various values of the damping.

2. Mathematical model

The mathematical model follows from the balance of kinetic and potential energies. Using Hamilton's principle and assuming the small deviate from the vertical, an approximate Lagrangian system in x, y-coordinates for components ξ, ζ can be obtained (see detailed derivation in (Náprstek and Fischer, 2009)):

$$\left.\begin{array}{rclr} \ddot{\xi} + \dfrac{1}{2r^2}\xi\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}(\xi^2+\zeta^2) + 2\beta_\xi\dot{\xi} + \omega_0^2\left(\xi + \dfrac{1}{2r^2}\xi(\xi^2+\zeta^2)\right) & = & -\ddot{a} & (a) \\ \ddot{\zeta} + \dfrac{1}{2r^2}\zeta\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}(\xi^2+\zeta^2) + 2\beta_\zeta\dot{\zeta} + \omega_0^2\left(\zeta + \dfrac{1}{2r^2}\zeta(\xi^2+\zeta^2)\right) & = & 0 & (b) \end{array}\right\} \quad (1)$$

The viscous damping has been introduced in a form of the Rayleigh function and denoted as β_ξ, β_ζ in respective directions. The natural frequency ω_0 of the corresponding linear pendulum is given by $\omega_0^2 = g/r$, where r is the suspension length of the pendulum and g is the gravitational acceleration. The external excitation of the suspension point $a(t) = a_0 \sin(\omega t)$ is assumed to be harmonic.

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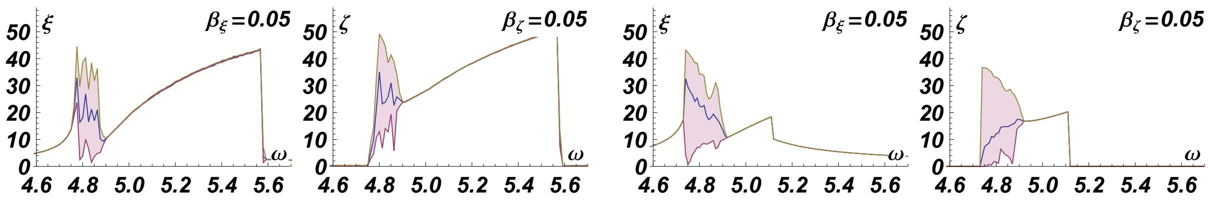

Fig. 1: Measured (two left plots) and computed (Eqn. (1), right pair of plots) resonance curves for symmetric damping $\beta_\xi, \beta_\zeta = 0.05$. In the each pair, longitudinal movement (ξ) is on the left and transversal response (ζ) on the right. Maximal, minimal and mean amplitudes are shown.

3. Experimental and numerical analysis

The experimental set-up is described in a high detail in (Pospíšil et al., 2011). Fundamental eigenfrequency of the pendulum was measured as $f_0 = 0.76$ Hz, i.e. $\omega_0 = 4.8\,\mathrm{rad}\cdot\mathrm{s}^{-1}$, its length was 0.41 m.

As can be seen in figure 1, the qualitative behaviour in the lower end of the resonance interval is rather comparable. On the other hand, a quite significant difference can be seen in the upper part of the studied frequency interval. It appears, that the experimental pendulum was able to follow the (less stable) upper branch of the solution during the sweep-up much better than the numerical solution. It is worth to mention, that for different values of damping the differences are not so dramatical.

The influence of individual damping coefficients on the overall response of the system in the both directions was studied numerically. The equation (1) was repeatedly integrated and the maximal amplitudes in both directions was recorded. The pair of plots in figure 2 shows response in longitudinal direction (ξ) on the left and transversal direction (ζ) on the right for $\omega = 4.8$. Values on the axes represent the damping coefficients $\beta_\xi, \beta_\zeta \in (0.005, 0.12)$. The colour map shows the distribution of the maximal amplitudes of xi and ζ in the left and right plot respectively. The dark colour indicates negligible or small amplitude of the response, whereas the bright colour shows the high response. The black dots in the each plot point at the discrete values of β used in simulation.

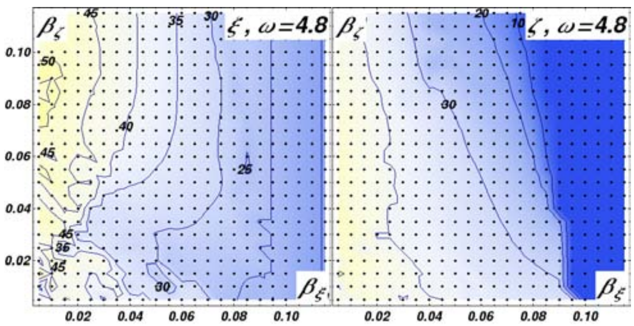

Fig. 2: Maximal amplitude of ξ (left) and ζ (right) depending on the values of damping coefficients in the both directions β_ξ, β_ζ.

4. Conclusions

It is shown and commented in the paper, that the presence of the spatial character of the system response does not depend significantly on the value of damping coefficient β_ζ. Similarly, the overall amplitude of the response is influenced mostly by β_ξ (longitudinal motion) and far less by β_ζ. The spatial response in the lower part of the resonance interval have higher amplitudes, but can be suppressed by smaller values of damping β_ξ. The lower amplitudes in the upper part of the resonance interval need higher damping β_ξ to be wiped off. The higher damping in transversal direction does not automatically mean the lower total response.

Acknowledgments

The support of the Czech Scientific Foundation No. 103/09/0094, Grant Agency of the ASCR No. A200710902 as well as the support of RVO: 68378297 and AV 0Z 2071913 are gratefully acknowledged.

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