18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 224–225
Paper #32

GENERALIZED LINEAR MODEL WITH AERO-ELASTIC FORCES VARIABLE IN FREQUENCY AND TIME DOMAINS

J. Náprstek, S. Pospíšil*

Abstract: *The paper demonstated the linkage of three models of the treatment of the structural aeroelastic instability using the double degree of freedom (DDOF) linear model. The frequent neutral models treating aero-elastic forces as certain constants are combined with the approach using flutter derivatives and indicial functions. The article fills the gaps in mathematical formulations. This approach allows formulate more sophisticated models combining main aspects of all groups in question. These models correspond by far better to results of wind channel and full scale measurements.*

Keywords: ***Flutter derivatives, Indicial functions, Non-symmetric systems, Dynamic stability.***

1. Introduction

Slender prismatic structures exhibited to strong dynamic wind effects (bridge decks, towers, chimneys, etc.) are frequently analyzed using a double degree of freedom (DDOF) linear model working with heaving and torsional components of a cross-section. This aero-elastic model is often adequate to study the system response until the first critical state is reached. Relevant mathematical models appearing in literature differ in principle by way of composition of aero-elastic forces. This criterion enables to sort them roughly in three groups. The first group can be possibly called neutral models - aero-elastic forces are introduced as suitable constants independent from excitation frequency and time. The second one involves flutter derivatives - they respect the frequency and flow velocity dependence of aero-elastic forces. The paper tries to make a synthesis of these groups on one common basis and to demonstrate their mutual linkage. In this study the bridge girder is considered as axially symmetric or almost symmetric with possible response components in heave u (vertical direction) and pitch φ.

2. Models of the aeroelastic forces

While neutral models are relatively the most simple and enable to provide many results analytically in a form of closed solution, the models with flutter derivatives have been introduced many years ago to investigate the stability in the aircraft, civil and other branches of engineering. They are introduced as functions in the frequency domain related to a particular cross-section without any link with other system parameters. Nevertheless, they can be understood as a certain extension of the damping and stiffness matrix elements. Flutter derivatives can also be interpreted as amplitudes Q or M of the heaving forces or the pitching moments, respectively, which should reach a unit amplitude of one response component under harmonic external kinematic excitation, while remaining components are kept zero in the same time. At the end, they are dimensionless functions of the excitation frequency ω, the stream velocity V and the geometric characteristic of the cross-section B $[m]$, written as follows:

$$\begin{aligned} Q(\omega) &= \mu_m V^2 \left(\tfrac{\mathrm{i}\omega B}{V}\kappa A_{11} + \kappa^2 A_{12}\right) U + \mu_m V^2 \left(\tfrac{\mathrm{i}\omega B^2}{V}\kappa A_{13} + \kappa^2 B A_{14}\right) \Phi, \quad \mu_m = \varrho/m \\ M(\omega) &= \mu_I V^2 B^2 \left(\tfrac{\mathrm{i}\omega}{V}\kappa A_{21} + \tfrac{1}{B}\kappa^2 B A_{22}\right) U + \mu_I V^2 B^2 \left(\tfrac{\mathrm{i}\omega B}{V}\kappa A_{23} + \kappa^2 A_{24}\right) \Phi, \quad \mu_I = 2\varrho/I \end{aligned} \tag{1}$$

where m or I are a mass or mass inertia moment of the vibrating body and ϱ is a specific mass of the air. $A_{ij}(\kappa)$ are the notification of the flutter derivatives assigned with respective forces. The definition itself of flutter derivatives apparently implicates that they can serve only to develop a linear mathematical

* Ing. Jiří Náprstek, DrSc., doc. Ing. Stanislav Pospíšil, Ph.D.
Institute of Theoretical and Applied Mechanics ASCR, v.v.i.; Prosecká 76, 190 00 Prague 9,
tel. +420.286 892 515, e-mail: naprstek@itam.cas.cz, pospisil@itam.cas.cz

model. Flutter derivatives can be incorporated into the governing equations only if these equations are expressed in the frequency domain. The governing differential system is written in a form of the two-way Laplace transform to unify the basis of the individual parts. Transformation exists if the system is stable and therefore influence of initial conditions disappears with increasing time, i.e. the steady state problems with the frequency $\omega = -\mathrm{i}\lambda$ can be investigated.

3. Stability analysis by generalized Routh-Hurwitz method

Dynamic stability of MDOF systems is closely related with eigen values of the characteristic matrix. If their real parts are all negative, the system is stable. To carry-out a general and careful analysis, a strategy based on an inspection of the characteristic polynomial $P(\lambda)$ is preferable. In such a case properties of the characteristic matrix are reflected in polynomial roots. So that limits separating their negative and positive real part values should be found. A large group of methods for searching these limits is based on properties of the Hurwitz matrix and its diagonal sub-determinants. Then the basic principle requests that all diagonal sub-determinants should be positive. Thus zero value of sub-determinants indicate individual stability limits. The characteristic polynomial of the system with the flutter derivatives A_{ij} represents the condition of the zero determinant of the related eigenvalue problem. In our case, provided the matrix of the system is multiplied by a factor λ^2, the condition of the stability gets a form of the characteristic equation of the eight degree of the parameter λ:

$$a_0\lambda^8 + a_1\lambda^7 + a_2\lambda^6 + a_3\lambda^5 + a_4\lambda^4 + a_5\lambda^3 + a_6\lambda^2 + a_7\lambda + a_8 = 0 \tag{2}$$

After some simplification, leading to the omitting of a_7, a_8 coefficients, the reduced ($n = 6$) characteristic equation occurs. Using the Routh-Hurwitz method, respective sub-determinants (stability conditions) read:

$$\begin{aligned}
\Delta_2 &= a_1 \cdot a_2 - a_0 \cdot a_3; \qquad \Delta_3 = a_3\Delta_2 - a_1^2 a_4 + a_0 a_1 a_5;\\
\Delta_4 &= a_4\Delta_3 - a_2(a_5\Delta_2 - a_1^2 a_6) + a_0(a_1 a_4 a_5 - a_0 a_5^2 - a_1 a_3 a_6);\\
\Delta_5 &= a_5\Delta_4 + a_1 a_5 a_6 \Delta_2 - a_3 a_6 \Delta_3 - a_1^3 a_6^2; \qquad \Delta_6 = a_6\Delta_5
\end{aligned} \tag{3}$$

These conditions of stability can be depicted both in the frequency domain $\omega_u \times \omega_\varphi$ and as the functions of the wind speed parameter V. The full version of the article illustrates this approach on an example.

4. Conclusion

Two types of double degree of freedom (DDOF) linear systems interacting with aero-elastic forces have been investigated and compared. The DDOF system under study describes inherent dynamic features of a slender prismatic beam attacked by a cross wind stream of a constant velocity (long bridge decks, guyed masts, towers, etc.). Two groups have been investigated: neutral models, where aero-elastic forces are introduced as suitable constants independent from excitation frequency and time and models using flutter derivatives for modelling the aero-elastic forces. The second group respect explicitly the stream velocity and the frequency of the system response. It succeeded to put both groups together on one common basis to demonstrate their linkage. The platform of qualitative investigation of aero-elastic critical states in a frequency plain has been significantly expanded with respect to the stream velocity. Memory effects ruling in aero-elastic DDOF system have been substantiated and compared in frequency and time domains. This approach can be used for the analysis of practical flow-structure interaction problems.

Acknowledgement

The support of the Czech Scientific Foundation No. 103/09/0094, Grant Agency of the ASCR No. A200710902 as well as the support of RVO 68378297 are gratefully acknowledged.

References

Pospíšil, S., Náprstek, J., (2011) General formulation of linear aeroelastic system stability and respective post-critical response types. In: *Proc. 13th Int. Conf. On Wind Engineering* (C. Geurts, B. Blocken eds)., Amsterdam, paper 177, 8 pgs.

Scanlan, R.H., Tomko, J.J. (1971) Airfoil and Bridge Deck Flutter Derivatives. *J. of Eng. Mech.* 97, 1717–1737.