

# **CRACK ANALYSIS IN MAGNETOELECTROELASTIC SOLIDS**

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**Abstract:** The paper discusses about crack analysis in magnetoelectroelastic solids. 2-D crack problems are considered. There are applied various electromagnetic boundary conditions on the crack-faces. The definition of the electromagnetic boundary conditions on the crack-faces plays an important role in the crack analysis of magnetoelectroelastic materials. Two extreme cases - the fully permeable and the fully impermeable crack surfaces are analysed. The finite element method is applied to solve crack boundary value problems. The coupling among magnetic, electrical and mechanical fields is adequately considered in magnetoelectroelastic solids subjected to external loads.

Keywords: magnetoelectroelastic composites, crack opening displacement

## 1. Introduction

Smart materials are widely used in practical engineering applications. It can be observed coupling effect between mechanical and electric fields (piezoelectric), mechanical and magnetic fields (piezomagnetic), mechanical and electric and magnetic fields (magnetoelectroelactic), etc. An electric potential is produced when a piezoelectric element is under stress or strain loading. This effect is called direct piezoelectric effect. When a mechanical deformation is produced by an electric field, it is a converse piezoelectric effect (Song at al., 2006). Magnetoelectric materials induce the polarization by a magnetic field, or conversely induce magnetization by an electric field Nan (1994). These materials are promising for a wide range of applications, such as four-state memories, magnetic field sensors and magnetically controlled optoelectric devices. It is important to analyze magnetoelectroelastic material for fracture resistance, because it is brittle. The electric and magnetic boundary conditions on the crack-faces are determined by the measure of shielding of the electric and magnetic fields. Thus, it is important to define the electromagnetic boundary conditions on the crack-faces. There are frequently considered two extreme cases. The first it is the fully permeable crack. This type does not shield the electric and magnetic field. The second one is fully impermeable crack, which shields the electric and magnetic field completely. The boundary value problems with cracks can be solved by several methods. For example finite element method (Enderlein at al., 2005), boundary element method (García-Sánchez et al., 2007) and meshless method (Sládek at al., 2008).

### 2. Basic equations of magnetoelectroelasticity

The coupling of the mechanical, electrical and magnetic fields in magnetoelectroelastic solids (Nan, 1994) is given by the following constitutive equations

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - e_{kij} E_k - d_{kij} H_k \tag{1}$$

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$$D_j = e_{jkl}\varepsilon_{kl} + h_{jk}E_k + \alpha_{jk}H_k$$
<sup>(2)</sup>

$$B_{j} = d_{jkl}\varepsilon_{kl} + \alpha_{kj}E_{k} + \gamma_{jk}H_{k}$$
(3)

The strain tensor  $\varepsilon_{ij}$ , electric field vector  $E_i$  and magnetic intensity vector  $H_i$  are related to independent variables - displacement, electrical potential and magnetic potential denoted by  $u_i$ ,  $\psi$  and  $\mu$ , respectively.  $\sigma_{ij}, D_i, B_i$  represent the stress tensor, the electric displacements, and the magnetic inductions, respectively. Material parameters are the elastic coefficients  $c_{ijkl}$ , dielectric permittivities  $h_{jk}$  and magnetic permeabilities  $\gamma_{jk}$ . Finally,  $e_{kij}$ ,  $d_{kij}$  and  $\alpha_{jk}$  are the coefficients for the piezoelectric, piezomagnetic, and magnetoelectric coupling, respectively.

#### 3. Numerical examples

The influence of the crack electromagnetic boundary conditions on the crack opening displacement  $u_3$  along the crack surface is shown in Fig. 1, when a combined loading is applied to the strip. A larger crack opening displacement appears in the case of fully impermeable electro-magnetic boundary conditions on the crack faces.



Fig. 1. Influence of the electromagnetic conditions on the crack displacement under a combined load.

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