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# HOMOGENIZED PHONONIC PLATES AND WAVE DISPERSION

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**Abstract:** *We consider the problem of wave propagation in periodically heterogeneous composite plates with high contrasts in elastic coefficients. The unfolding method of homogenization is applied to obtain limit plate models. Due to the high contrast ansatz in scaling the elasticity coefficients of compliant inclusions, the dispersion properties are retained in the limit when the scale of the microstructure tends to zero. We study two plate models based on the Reissner-Mindlin theory and on the Kirchhoff-Love theory. We show that, when the size of the microstructures tends to zero, the limit homogeneous structure presents, for some wavelengths, a negative "mass density" tensor. This means that there exist intervals of frequencies in which there is no propagation of elastic waves, the so-called band-gaps.*

**Keywords:** ***phononic materials, plate models, homogenization, band gaps, wave dispersion***

## 1. Introduction

We consider problems of wave propagation in periodically heterogeneous plates with high contrasts in elastic coefficients. Following the approach of Ávila et al. (2008) and Rohan et al. (2009) we apply the unfolding method of homogenization (Cioranescu et al., 2008) to obtain limit plate models. Two cases are studied: 1) according to the Reissner-Mindlin theory the plate deformation is described by the mid-plane deflections and by rotations of the plate cross-sections which account for the shear stress effects; 2) using the Kirchhoff-Love theory, the plate deflections are described by the bi-harmonic operator, thus neglecting the shear effects. In both cases we assume such heterogeneities which depend on the mid-plate coordinates only, but do not change with the transversal coordinate. As an example we can consider plates with soft cylindrical inclusions. Under such restrictions the homogenization is applied to the plate equations with the elastic coefficients defined as periodically fluctuating functions associated with the heterogeneities. Due to the high contrast ansatz in scaling the elasticity coefficients of inclusions, as employed in Ávila et al. (2008); Rohan and Miara (2011); Cimrman and Rohan (2010), dispersion properties are retained in the homogenized model when the scale of the microstructure tends to zero.

We show that, when the size of the microstructures tends to zero, the limit homogeneous structure presents the phononic effect: for some wavelengths, a "mass density" tensor can be negative, see Rohan and Miara (2011). This means that there exist intervals of frequencies in which there is no propagation of elastic waves, the so-called band-gaps.

## 2. Homogenized plates

The standing waves propagating in the homogenized plate of the Reissner-Mindlin type are described in terms of amplitudes $(\boldsymbol{\theta}, w) \in \mathbf{H}_0^1(\Omega) \times \in H_0^1(\Omega)$ which satisfy the following equations:

$$
\begin{aligned}
& -\omega^2 \int_\Omega \left( \frac{h^3}{3} [\boldsymbol{\mathcal{M}}(\omega^2)\boldsymbol{\theta}] \cdot \boldsymbol{\psi} + h\mathcal{N}(\omega^2) w z \right) \\
& + \frac{h^3}{3} \int_\Omega [\boldsymbol{\mathcal{D}} \boldsymbol{e}_x(\boldsymbol{\theta})] : \boldsymbol{e}_x(\boldsymbol{\psi}) + h \int_\Omega [\boldsymbol{\mathcal{G}}(\nabla_x w - \boldsymbol{\theta})] \cdot (\nabla_x z - \boldsymbol{\psi}) = 0
\end{aligned}
\tag{1}
$$

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for all $\boldsymbol{\psi} \in \mathbf{H}_0^1(\Omega), z \in H_0^1(\Omega)$ where $\boldsymbol{\mathcal{D}}$ is the 4th order tensor of homogenized elasticity coefficients, $\boldsymbol{\mathcal{G}}$ is the 2nd order tensor describing the shear stiffness of the plate, $\boldsymbol{\mathcal{M}}(\omega^2)$ and $\mathcal{N}(\omega^2)$ are homogenized mass coefficients, both depending on a given frequency $\omega^2$ of the incident waves.

In analogy we derive the model of standing waves in the homogenized plate of the Kirchhoff-Love type: the defelection amplitude $w \in H_0^2(\Omega)$ satisfies the following equation

$$-\omega^2 h \int_\Omega \bar{\rho} w v - \omega^2 \frac{h^3}{3} \int_\Omega (\boldsymbol{\mathcal{M}}(\omega^2) \nabla w) \cdot \nabla v + \frac{h^3}{3} \int_\Omega (\boldsymbol{\mathcal{D}} \nabla\nabla w) : \nabla\nabla v = 0 \qquad (2)$$

for all $v \in H_0^2(\Omega)$ where $\bar{\rho}$ is the average density of the composite, $\boldsymbol{\mathcal{D}}$ is the 4th order homogenized bending stiffness tensor and $\boldsymbol{\mathcal{M}}(\omega^2)$ is the homogenized mass tensor.

## 3. Conclusions

We present homogenized models of wave propagation in strongly heterogeneous plates, considering the Reissner-Mindlin (R-M) and the Kirchhoff-Love (K-L) theories. The homogenization results reveal dispersion properties for the homogenized plates: we claim that there exist bands of frequencies for which the wave equations admit evanescent solutions only, at least for certain polarizations. There is remarkable difference between the R-M and K-L models: while for R-M the wave polarization is determined by components of $(\boldsymbol{\theta}, w)$, i.e. the rotation and deflection, for K-L there is just a scalar wave associated with the deflection $w$.

The phononic effect, in general, is associated with vibration modes excited at the “microscopic” level. These modes determine “positivity”, or “negativity” of the homogenized masses $\boldsymbol{\mathcal{M}}(\omega^2)$ and $\mathcal{N}(\omega^2)$; in Ávila et al. (2008) we described how this observation can be employed to predict band gaps. The classical method of the band gap identification is based on analysis of guided waves, thus, upon construction of dispersion curves; it is necessary to compute frequencies for selected wave numbers ranging the Brillouin zone, cf. Rohan et al. (2009).

Further research will be focused to study dispersion properties and band gaps distributions for some basic microstructures. An important restriction of both presented models is related to the transversal isotropy: here only cylindrical inclusions are admissible, although their shapes can be arbitrary. To treat more general composite plates with e.g. spheroidal inclusions, the homogenization procedure must be applied to a 3D composite with thickness proportional to $\varepsilon$, i.e. to the microstructure scale.

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