18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 34–35
Paper #140

COMPLEX MODEL OF THE LOWER URINARY TRACT

M. Brandner [*], J. Egermaier [**], H. Kopincová [***], J. Rosenberg[****]

Abstract: *The complex model of the lower part of the urinary tract is introduced. It consists of the detrusor smooth muscle cell model and the detailed 1D model of the urethra flow. For the modelling calcium dynamics the approach from Koenigsberger at al. (2008) was adopted. The model includes the ATP consumption calculation according to Hai and Murphy (1992). The main part is devoted to the development of a simple bladder model and the detrusor contraction during voiding together with the detailed model of the urethra flow.*

Keywords: ***urinary tract, bladder, urethra fluid flow, steady state preserving***

1. Introduction

The voiding is a very complex process. It consists of the transfer of information about the state of the bladder filling in to the spinal cord. Next part is the sending of the action potentials to the smooth muscle cells of the bladder. The smooth muscles have a lot of different forms in contradiction with the striated muscles. The sliding between actin and myosin causing the change of the form (length) of the muscle cell and its stiffness can be observed as a kind of growth and remodeling. This approach described e.g. in Rosenberg and Hynčík (2008) is used in this model. The calcium dynamics and the phosphorilation is modeled very precisely in the work Hai and Murphy (1992). To be able to describe the very complex processes in the SMC in the efficient form it is necessary to use the irreversible thermodynamics. Using all this approaches it was developed the algorithm published in Rosenberg (2011b). In this contribution we join on the results of this paper. The simple model of the whole bladder and the detailed 1D model of the urethra flow is added. Some examples of the numerical experiments are shown.

2. Bladder and voiding model

Because the whole model of the smooth muscle contraction is described in the mentioned paper we would like only to introduce some further used variables. The flux of the mediator in synapse $J_{agonist}$, the shortening of the muscle fiber $x \cdot y$ where the variables x and y are given by the set of equations.

To model the contraction of the bladder during the voiding process we will use the very simple model according Laforet and Guiraud (2007) and Arts at al. (1991). For the pressure in the bladder gilts

$$p = \frac{V_{sh}}{3V} \cdot \tau, \qquad \tau = \frac{F}{S}, \tag{1}$$

where V_{sh} is the volume of the bladder wall, V the inner volume, τ stress in the muscle fibre, S the inner surface and F the force in the muscle. Using the formulas for the isotonic contraction, we obtain after short calculation the formula for the pressure $p(q, x \cdot y)$, which will be used further.

[*]Doc. Ing. Marek Brandner, Ph.D.: NTIS New Technologies for Information Society, University of West Bohemia in Pilsen, Univerzitni 8; 306 14, Pilsen; Czech republic, e-mail: brandner@kma.zcu.cz
[**]Ing. Jiří Egermaier, Ph.D.: Department of Mathematics, University of West Bohemia in Pilsen, Univerzitni 8; 306 14, Pilsen; Czech republic, e-mail: jirieggy@kma.zcu.cz
[***]Ing. Hana Kopincová, Ph.D.: NTIS New Technologies for Information Society, University of West Bohemia in Pilsen, Univerzitni 8; 306 14, Pilsen; Czech republic, e-mail: kopincov@kma.zcu.cz
[****]Prof. Ing Josef Rosenberg, DrSc.: Department of Mechanics, University of West Bohemia in Pilsen, Univerzitni 8; 306 14, Pilsen; Czech republic, e-mail: rosen@kme.zcu.cz

3. Urethral flow

The fluid flow through the urethra is described by the system based on model in Stergiopulos at al. (1993)

$$\begin{aligned} a_t + q_x &= 0, \\ q_t + \left(\frac{q^2}{a} + \frac{a^2}{2\rho\beta}\right)_x &= \frac{a}{\rho}\left(\frac{a_0}{\beta}\right)_x + \frac{a^2}{2\rho\beta^2}\beta_x - \frac{q^2}{4a^2}\sqrt{\frac{\pi}{a}}\lambda(Re), \end{aligned} \tag{2}$$

where $a = a(x,t) = (p - p_e)\beta + a_0$ is the cross-section area, $q = q(x,t)$ is the flow rate, ρ is the fluid density, $a_0 = a_0(x)$ is the cross-section of the tube under no pressure, $\beta = \beta(x,t)$ is the coefficient describing tube compliance and $\lambda(Re)$ is the Mooney-Darcy friction factor ($\lambda(Re) = 64/Re$ for laminar flow).

The discretization of the system (2) is based on the extension by other equations so that the augmented vector of unknown functions is $[a, q, av^2 + \frac{a^2}{2\rho\beta}, \frac{a_0}{\beta}, \beta]^T$. This was derived in detail in Brandner at al. (2009). The advantage of this step is in the conversion of the nonhomogeneous system to the homogeneous one. Furthermore, it is possible to construct the positive semidefiniteness high-resolution scheme, which preserves general steady states of the fluid flow in the urethra.

4. Conclusion

We presented the complex model of the lower part of the urinary tract. It was developed a simple bladder model and the detrusor contraction model during voiding together with the detailed model of urethra flow. The urethra flow was described by the high-resolution positive semidefiniteness method, which preserves general steady states. For the practical application the identification of the parameters is necessary.

Acknowledgments

This work was supported by the European Regional Development Fund (ERDF), project NTIS - New Technologies for Information Society, European Centre of Excellence, CZ.1.05/1.1.00/02.0090 and the project SGS-2010-077 Support of Biomechanics at the faculty of applied sciences, University of West Bohemia in Pilsen.

References

Arts, T., Bovendeerd, P.H.M., Prinzen, F.W., Reneman, R.S. (1991), Relation between left ventricular cavity pressure and volume and systolic fiber stress and strain in the wall. In: *Biophys. J.*,1991, vol. 59, p. 93-102

Brandner, M., Egermaier, J., Kopincová, H. (2009), Augmented Riemann solver for urethra flow modelling. In: *Mathematics and Computers in Simulations*, Vol. 80, No. 6, pp. 1222-1231

Hai, C.M. and Murphy, R.A. (1992), Adenosine 5'-triphosphate consumption by smooth muscle as predicted by the coupled four-state crossbridge model. In: *Biophysical Journal*,1992, vol. 61, no. 2, p. 530-541

Koenigsberger, M., Sauser, R., Seppey, D., Beny, J.-L., Meister, J.-J. (2008), Calcium dynamics and vosomotion in arteries subject to isometric, isobaric and isotonic conditions. In: *Biophysical Journal* Vol. 95, 2008 p. 2728-2738

Laforet, J. and Guiraud, D. (2007), Smooth Muscle Model For Functional Electric Stimulation Applications. In: *Proceedings of the 29th Annual International Conference of the IEEE EMBS*, Cite Internationale, Lyon. France August 23-26. 2007

Rosenberg, J. (2011b), Modelling of the voiding process. In: *Extended abstracts, 27th Conference with international participation Computational Mechanics 2011 November 7.-9.*, 2011, Plzeň.

Rosenberg, J. and Hynčík, L., (2008), Modelling of the influance of the stifness evolution on the behaviour of the Muscle fibre. In: *Human Biomechanice 2008*, International Conference, 29.9.-1.10.2008 Praha, Czech Republic

Stergiopulos, N., Tardy, Y., Meister, J.-J. (1993), Nonlinear Separation of Forward and Backward Running Waves in Elastic Conduits. In: it Journal of Biomechanics, Vol. 26, pp. 201-209