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FINITE ELEMENT CONTACT-IMPACT ALGORITHM IN EXPLICIT TRANSIENT ANALYSIS

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Abstract: *This work addresses three issues in computational modelling of contact-impact problems: i) overviews a contact algorithm proposed by these authors, ii) local search treatment based on the modification of the Nelder-Mead simplex method, iii) discusses an algorithmic aspects of contact algorithm in conjunction with the explicit time integration scheme. The talk closes with the presentation of several numerical examples including the longitudinal impact of two thick plates, for which analytical solution is available.*

Keywords: ***FEM, contact-impact, explicit dynamics, local contact search***

1. Introduction

In the context of the finite element method, a frictionless three-dimensional contact-impact algorithm using pre-discretization penalty formulation was proposed (Gabriel et al., 2004). The key feature of this algorithm is that the local search and the penalty constraint enforcement are performed on the Gausspoint level of linear/quadratic serendipity elements rather than the nodal level of a finite element mesh. The method is shown to be consistent with the variational formulation of a continuum problem, which enables incorporation of higher-order elements with midside nodes to the analysis. Owing to a careful description of kinematics of contacting bodies when the non-linearized definition of penetration has been introduced, the displacement increments in the course of one load step are permitted to be large. Thus, the extension to geometrically nonlinear problems is straightforward. The algorithm proves to be robust, accurate and symmetry preserving—no master/slave surfaces have been introduced.

In proposed algorithm the local search represents measuring penetration of a Gauss point through the counterpart's object surface. It is necessary first to define the outward normal and then to compute its intersection with a curved surface, establishing distance. Although appearing trivial at first glance the numerical solution process is far from being easy, especially when dealing with severely distorted surfaces. In Ref. (Gabriel et al., 2010) several methods for the solution of non-linear algebraic systems were thoroughly tested: the Newton-Raphson method, the least square projection, the steepest descent method, Broyden's method, BFGS method and the simplex method. The effectiveness of these methods was performed by means of the benchmark configuration of distorted contact segment from static solution of bending of two rectangular plates over a cylinder (Gabriel et al., 2004). The most fitting method turned out the modification of the Nelder-Mead simplex method (Nelder and Mead, 1965), which belongs to very popular and simple direct search technique that has been widely used in unconstrained optimization problems.

In this paper, we focused on the performance of the Nelder-Mead simplex method for local contact search treatment in dynamic contact-impact problem. First, the formulation of a closest point projection problem is presented in Section 2. The idea of the Nelder-Mead method is outlined in Section 3. Finally, the effectiveness of contact algorithm with implemented Nelder-Mead simplex method for contact search procedure is demonstrated by test example of the longitudinal impact of two thick plates in Section 4.

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2. Formulation of the closest point projection

Let us consider the slave quadrature point $\mathbf{y}_s \in \mathbb{E}^3$ and the master segment γ_c. The aim of the local contact search is to calculate the parametric coordinates $\xi_1, \xi_2 \in [-1, 1]$ corresponding to projection $\bar{\mathbf{y}}(\xi_1, \xi_2) \in \mathbf{E^3}$ of the quadrature point $\mathbf{y}_s$ (see Fig. 1).

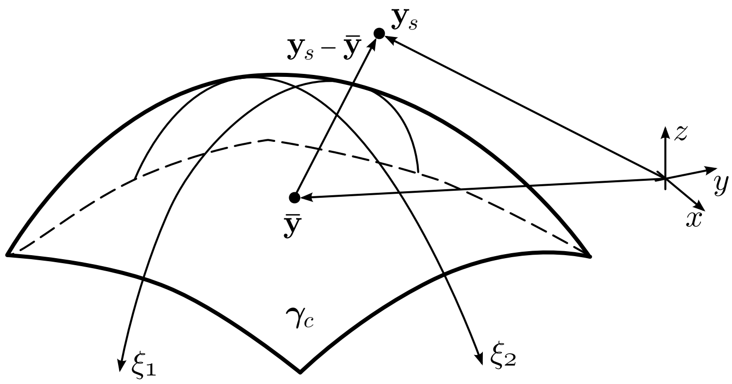

Fig. 1: Formulation of the minimization problem

Such a point has to satisfy

$$\bar{\mathbf{y}} = \min_{\mathbf{y}\in\gamma_c} \{(\mathbf{y}_s - \mathbf{y}) \cdot (\mathbf{y}_s - \mathbf{y})\} \tag{1}$$

where the minimization of the inner product on $\mathbb{E}^3$ instead of more natural Euclidean norm has been used. Hence, the minimized function is defined as

$$f = (\mathbf{y}_s - \mathbf{y}) \cdot (\mathbf{y}_s - \mathbf{y}) \tag{2}$$

The necessary condition for local extremum is

$$\begin{aligned} (\mathbf{y}_s - \mathbf{y}) \cdot \frac{\partial \mathbf{y}}{\partial \xi_1} &= 0, \\ (\mathbf{y}_s - \mathbf{y}) \cdot \frac{\partial \mathbf{y}}{\partial \xi_2} &= 0 \end{aligned} \tag{3}$$

The master segment γ_c is parametrized by

$$\mathbf{y}(\xi_1, \xi_2) = \sum_{i=1}^{n} N_i(\xi_1, \xi_2)\, \mathbf{Y}_i \tag{4}$$

where $N_i(\xi_1, \xi_2) : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are the shape functions, n is the number of nodes and $\mathbf{Y}_i \in \mathbb{E}^3$ are the global coordinates of nodes. Note that the partial derivations are constant and Eqn. (3) is system of linear equations for linear triangular segments. If higher order elements are taken into account, Eqn. (3) results in the system of non-linear algebraic equations. The inequality constraints $|\xi_1|, |\xi_2| \leq 1$ for isoparametric segment γ_c are not explicitly imposed. The solution of the unconstrained problem lying outside the permissible range indicates that the quadrature point does not penetrate onto the master segment.

3. Nelder-Mead simplex method

Let us consider the minimization of the function f (2). The points $\mathbf{x}_i^k,\ i = 1, 2, 3$ define the current simplex in two-dimensional space. We set up

$$\mathbf{x}_h^k = \arg\max_i \left(f\left(\mathbf{x}_i^k\right)\right), \tag{5}$$

$$\mathbf{x}_l^k = \arg\min_i \left(f\left(\mathbf{x}_i^k\right)\right) \tag{6}$$

as the points with maximum and minimum function value, respectively. Further, we define

$$\bar{\mathbf{x}}_i^k = \frac{\sum_{\substack{i=1 \\ i\neq h}}^{2} \mathbf{x}_i^k}{2} \tag{7}$$

as the center of the points $\mathbf{x}_i^k$ with $i \neq h$. At each stage $\mathbf{x}_h^k$ is replaced by a new point. In Ref. (Nelder and Mead, 1965), three operations are applied: reflection, contraction, and expansion. In our modification only the reflection is considered. It is defined by

$$\mathbf{x}^* = \bar{\mathbf{x}}_i^k + \alpha \left(\bar{\mathbf{x}}_i^k - \mathbf{x}_h^k\right), \tag{8}$$

where α is the reflection coefficient (positive constant). We choose α simply equal to one. Thus, the simplex preserves regularity. In case that $f(\mathbf{x}^*) \geq f(\mathbf{x}_h^k)$ the vertex with second highest value is reflected instead of $\mathbf{x}_h^k$. When one of the vertices has still the same position, it indicates that the simplex rotates above a local extremum. Therefore, the simplex edge length a is halved after m iterations. The number of iteration m can be estimated by the empiric formula

$$m = 1.65n + 0.05n^2, \tag{9}$$

where $n = 2$ for two-dimensional case.

4. Longitudinal impact of two plates

The longitudinal impact of two thick plates was studied, for which the analytical solution was available (Brepta and Valeš, 1987). Despite the problem is two-dimensional one it could be used for testing different methods for three-dimensional local contact search. The plates dimensions were: thickness $2d = 5$ mm, length 2.5 mm. Young's modulus, Poisson's ratio and density, respectively, were $E = 2.1 \times 10^5$ MPa, $\nu = 0.3$, $\rho = 7800\,\mathrm{kg/m^3}$. The plates made contact with initial velocity $v_0 = 1$ m/s prescribed at time $t = 0$ s (Fig. 2).

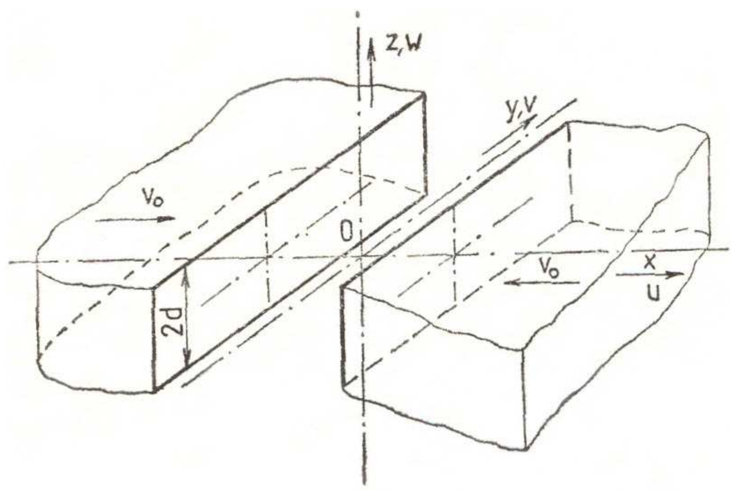

Fig. 2: Longitudinal impact of two plates

The analytical solution (Brepta and Valeš, 1987) utilizing the Laplace transform is rather complex. The distributions of displacements and stresses are cast in the form of infinite series of improper integrals which are evaluated numerically. For illustration, theoretical positions of wave fronts for a short time after impact are plotted in Fig. 3. At the instant the faces of the plates come into contact there are aroused elementary dilatation waves at all points of the contact area. The envelope of these waves is represented by a wave with a plane wave front, propagating in both directions at speed of dilatation waves c_1. From the boundary points A, D of the contact area emanates a reflected wave which continues propagating in perpendicular direction to x, y plane at speed c_1. Behind the dilatation wave the transversal waves proceeds at speed c_2. In the region bounded by plane wave fronts of the dilatation wave and by circular wave fronts of the wave starting from the points A and D, the state of stress is the same as that encountered by a longitudinal impact of half-spaces.

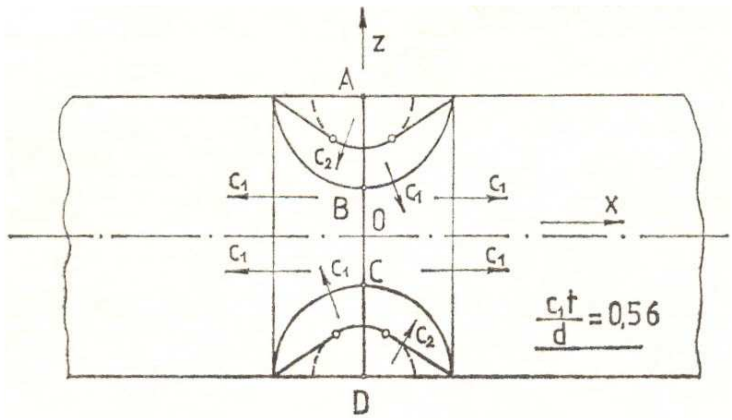

Fig. 3: Theoretical position of wave fronts for $c_1 t/d = 0.56$ after (Brepta and Valeš, 1987)

In view of symmetry, only one half of the plates was discretized using 100×100 eight-node linear brick elements per each plate. For the integration of equilibrium equations, the central difference with the lumped mass matrix was employed. The time step was chosen very small corresponding to the dimensionless Courant number C$o = 0.125$.

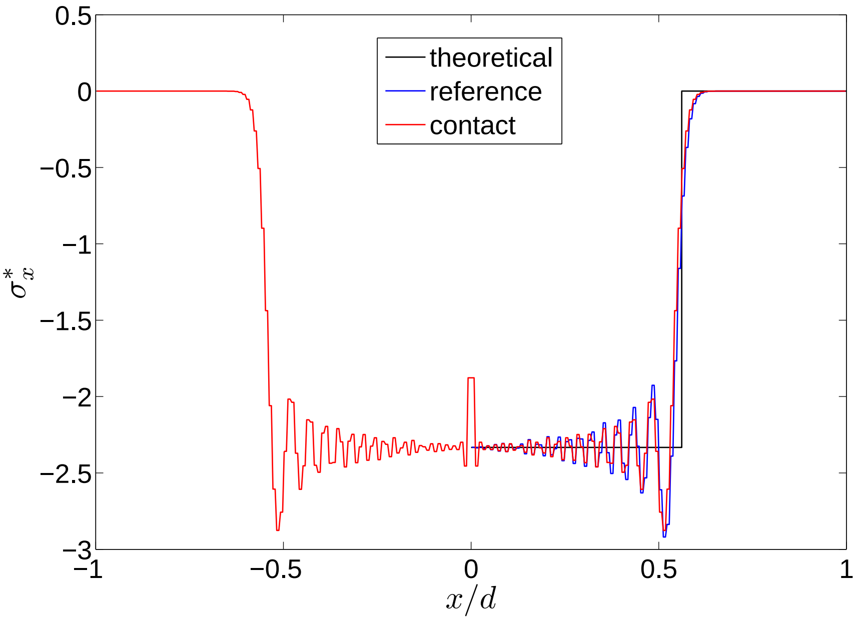

Fig. 4: Longitudinal stress distribution σ_x^* along x-axis for $z/d = 0$

The normalized longitudinal stress distribution $\sigma_x^* = \sigma_x c_1/\Lambda v_0$ (Λ is Lamé's constant) along x-axis is drawn in Fig. 4. The results are plotted for normalized time $c_1 t/d = 0.56$ and coordinate $z/d = 0$, for which no reflections from boundaries occur. Except the contact analysis a symmetric reference calculation was performed, where the longitudinal displacements of the front-end nodes of the plate were fixed.

In Fig. 4 the contact solution is plotted by red line while the solution based on the reference calculation is denoted by blue line. In addition, the theoretical solution corresponding to uniaxial strain condition is plotted by the black line. Quite a good agreement between the contact and reference calculation was observed. It should be emphasized that the symmetry of longitudinal stress distributions was perfectly preserved in contact analysis. Thus, the capability of the Nelder-Mead simplex method implemented in local search procedure was confirmed. It is clear that the numerical solution was influenced by dispersion errors caused by both FE spatial and time discretization. In comparisons with the continuum solution the speed of the longitudinal wave was slower. This fact follows from the theoretical dispersion diagrams derived in Ref. (Plešek et al., 2010).

The normalized transversal stress distribution $\sigma_z^* = \sigma_z c_1/\Lambda v_0$ along z-axis is drawn in Fig. 5. In contrast to graphs in previous Fig. 4 these distributions are strongly influenced by the longitudinal and transversal waves reflected from the boundary of plate. Before the arrival of these waves the solution is identical to the constant values $\sigma_z^* = -1$ corresponding to a half-space impact problem. It should be pointed out that the accuracy of analytical solution is strongly influenced by the number of terms included in the series of improper integrals (Brepta and Valeš, 1987). The analytical solution plotted in this paper was derived from the summation of the first 300 terms of this series.

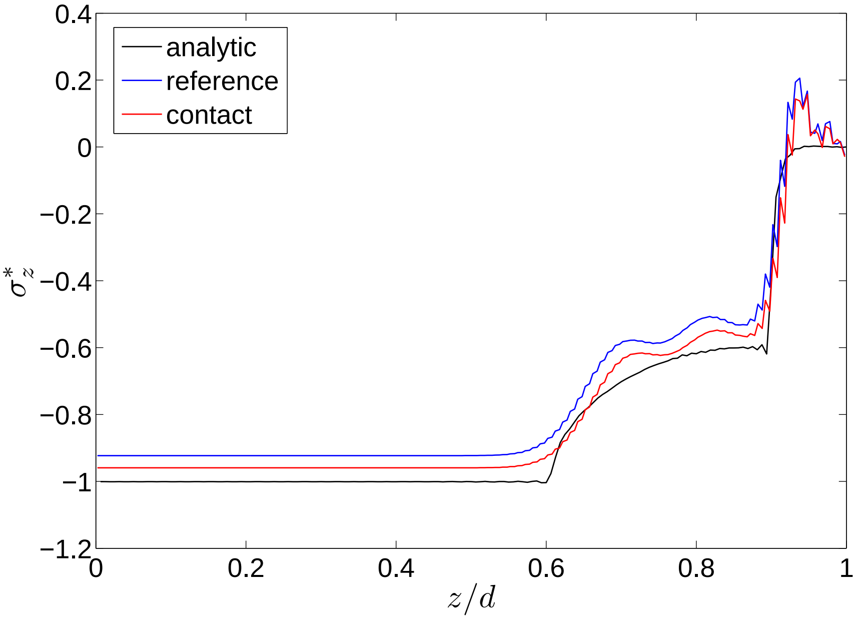

Fig. 5: Transversal stress distribution σ_z^* along z-axis for $x/d = 0.4$

Acknowledgments

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