18th International Conference
ENGINEERING MECHANICS 2012
Svratka, Czech Republic, May 14 – 17, 2012

pp. 206–207
Paper #179

COMPUTATIONAL HOMOGENIZATION OF ACOUSTIC PROBLEM IN PERFORATED PLATES

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Abstract: *We consider acoustic wave propagation described by Helmholtz equation and involving homogenized transmission conditions imposed along a thin perforated interface separating two halfspaces occupied by the acoustic medium. The homogenized transmission conditions are imposed on this perforated interface. The transmission conditions were obtained as the two-scale homogenization limit of the standard acoustic problem imposed in the layer perforated by a sieve-like obstacle with periodic structure. By using the sensitivity analysis we can solve the problem of an optimal design of the perforation to minimize the transmission loss in a domain embedding the interface. The perforated periodic structure is represented by a reference computational cell, whereby its geometry is controlled by the spline functions.*

Keywords: ***linear acoustics, homogenization, sensitivity analysis, transmission condition***

1. Introduction

Optimization of noise transmission in the acoustic fluid belongs to important merits of the acoustic engineering. Sieve-like structures are classical elements employed in noise-reducing devices. For example, in the exhaust silencers of the combustion engines the gas flows through ducts equipped with various sieves which in part may influence the transmission losses associated with acoustic waves propagating in the exhaust gas. In aerospace and automotive industry there are many applications related to acoustic waves and fluid flow where optimal design of the sieves (perforated slabs) is a challenging problem.

In the paper we deal with the acoustic transmission through a *perforated interface*, cf. Chen (1996); Bonnet-Bendhia and others (2005). The transmission conditions to be imposed on the interface plane were derived in Rohan and Lukeš (2010), using the asymptotic analysis. The limit model of an interface plane involves some homogenized impedance coefficients depending on the so-called microscopic problems; these are imposed in the *reference periodic cell* embedding an obstacle which represents the perforation. The two-scale modeling approach allows for an efficient treatment of complicated designs of perforations. The limit model was subjected to the sensitivity analysis in Rohan and Lukeš (2009). It resulted in the sensitivity formulas for the homogenized coefficients and we obtained the total variation of an objective function depending on the acoustic pressure w.r.t. the obstacle shape at the "microlevel".

An abstract optimization problem is formulated at three levels: at the "global" one the pressure field is controlled by an interface variable – the transversal acoustic momentum involved in the homogenized transmission condition; at the "homogenized interface" level, the interface variables are satisfy the non-local transmission conditions depending on the homogenized impedance parameters; finally, at the "microscopic level" these impedance (homogenized) parameters depend on solutions of auxiliary local problems featured by the shape of perforations.

2. Optimal design problem

We consider the problem of the wave propagation in a waveguide filled by the acoustic medium which is subdivided by perforated plane Γ_0 in two disjoint subdomains, see Fig. 1. The frequently used criteria of optimality in acoustics is related to transmission loss (TL) evaluated using input and output pressures.

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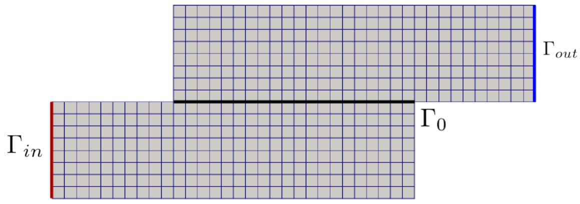
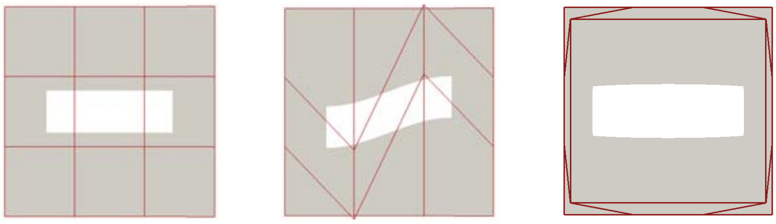


Fig. 1: Acoustic waveguide equipped with a perforated plate Γ_0.

The perforated plane is designed by repeating a reference cell which is "perforated" by the rigid obstacle; its geometry is controlled by spline functions. In our optimal design problems we use four inner control points to control the obstacle shape, see Fig. 2, so eight optimization parameters are associated in 2D (two coordinates for each control point). The objective function is defined to achieve by its minimization a desired (target) value of the transmission loss $TL(p)$ is close to a required value $\widetilde{TL}$. The results (just local minima guaranteed) were obtained by the SQP algorithm with box constraints which secure the "mesh deformation" during the design iterations. We started optimizing with two different initial shapes of the obstacle (Fig. 2(A,B)); in both cases, the optimization process resulted in the shape depicted in Fig. 2(C).



(A) (B) (C)

Fig. 2: (A) and (B): two different initial states used in optimization; (C): final shape of the obstacle after optimization.

3. Conclusion

The "multi-scale" homogenization approach is employed for an efficient treatment of the optimal perforation design. We use the spline parametrization to control the shape of the solid obstacle forming the perforation. The model and its sensitivity are implemented in our in-house developed finite element based code *SfePy* (Cimrman and others (2012)).

Acknowledgments

This work was supported by the European Regional Development Fund (ERDF), project "NTIS – New Technologies for Information Society", European Centre of Excellence, CZ.1.05/1.1.00/02.0090 and by Czech Science Foundation, project GA101/12/2315.

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