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MATERIAL NON-LINEAR BEAM ELEMENT WITH SHEAR CAPACITY

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Abstract: *The paper describes a new formulation of beam elements for deformational variant of FEM, which respects non-linear material behaviour. This formulation considers combination of shear and axial loading and is suitable for short or torque beams similarly to the Timoshenko theory. Non-uniform warping and influence of transversal contraction are not considered in the formulation. The cross section can be of arbitrary known shape and composed of more materials. The presented element respect real distribution shear stress over cross section taking material non-linearity into consideration*

Keywords: ***beam element, FEM, material non-linearity, shear, torque***

1. Introduction

More economical usage of materials at technical objects is connected with the development of simulation tools. A similar situation is at beam construction which is abundantly represented in technical practice. Just use of material non-linearity provides reserves in material usage. Besides that in practice more and more emphasis is placed on robustness of simulation tools without neccessarity of deep knowledge of service. Frequent use of some materials makes correct use of linear models impossible, for example reinforced concrete. For these reasons, based on practical requests of users, the following formulation of beam element was developed. Geometry of the presented element is shown on fig. 1.

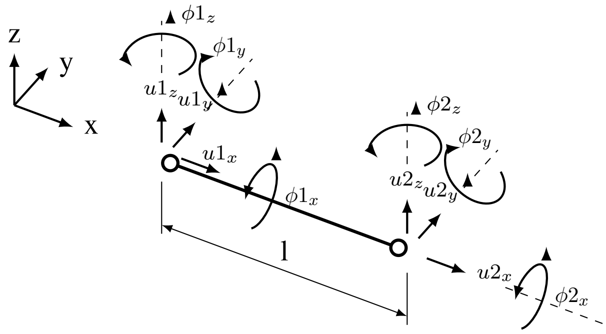

Fig. 1: Beam element

During making solution about used formulation of the element that should be implemented an extensive research of available formulations in commercial CAE software's (Nastran, Abaqus, Marc, Ansys) was done. All softwares provides a lot of formulations of the beam element including material non-linearity. Their definition it is possible to find for example here Crisfield (2000),Němec (2010) and Zienkiewicz (2000). Detail assessment shows that there is provided robust and sufficient solution for Euler formulation of the element for uniaxial stress in all software's. But in case of shear stress the situation is dismal. Here are provided solutions for Timoshenko formulation as well but detail analysis proves that these solutions are very simplified. For example Ansys provides the element BEAM 188/189. This element however supposes constant shear strain over cross section and strain corresponding to linear torsion. But real stress distribution at shear loading is different (for example Grutmann (1999)). In other

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CAE software's the situation is similar. Even literature search did not provide more complex formulation for material non-linearity.

None of available formulations of the beam element respect real distribution shear stress over cross section taking material non-linearity into consideration.

Solution of plastic torque is known for long time (for example here Chakrabarty (2006)). Numeric solution of plastic torque for an arbitrary cross section is defined here Gruttmann (2001). By small modification the author extend this solution to general shear loading Kabeláč (2011). The model for axial part of stress is generally known (good description in Crisfield (2000)). By combining of these solutions we obtain real stress distribution over cross section at arbitrary combination of beam loading. In that way formulated cross section behaviour is combined with appropriate shape function. Result of this combination is here presented formulation of beam element, which respects real shear stress distribution over arbitrary cross section.

Use of various cross section characteristics, specified by an user, increases demands on an user and is a source of possible mistakes. Therefore we left numerical values as input and the only input is a shape of cross section or FEM mesh of cross section. On this mesh more materials can be defined. It means the model is also usable for composite cross section.

The represented model is valid for small strain, straight prismatic beam and free warping of cross section. The influence of transversal contraction is not considered. In connection to co-rotational formulation its involving in geometric non-linearity is easy.

2. Conclusions

The element presented here is robust enough for practical use. Cross section can have arbitrary origin point. It can be composed of more materials and the formulation is independent on material model. A disadvantage of the element is neglecting of non-uniform warping especially in prevailing shear loading. The element will be tested in future and it is supposed to be implemented in commercial CAE software.

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