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NUMERICAL ANALYSIS OF COUPLED HEAT AND MOISTURE TRANSFER BASED ON KUNZEL MODEL

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Abstract: *Coupled heat and moisture transfer is still more often used in many civil engineering problems. In connection with concrete and plasters, the Künzel model is very popular. Unfortunately, very different orders of material parameters have devastating influence on the condition number of matrices obtained after space and time discretization of problems. It results in severe numerical difficulties. This contribution deals with some strategies leading to better numerical behaviour of the coupled transport processes.*

Keywords: ***coupled heat and moisture transport, Künzel model, condition number, non-symmetric systems of equations.***

1. Introduction

The material coefficients depend on the actual values of temperature and relative humidity and they are not constant. It means, the conductivity matrix of material has to be computed in every time step. In some configurations, the conductivities are very small and it leads to serious numerical problems because there are zero diagonal matrix entries. In such cases, appropriate degrees of freedom should be removed from the system and they can be returned back when the conductivities become physically important.

2. Künzel model of coupled heat and moisture transport

The Künzel model of coupled heat and moisture transfer in buildings or building components leads to the system of mass and heat balance equations in the form

$$\frac{\partial w}{\partial t} = \mathrm{div}\left((D_\varphi + \delta_p p_{vs}) \nabla\varphi + \delta_p \varphi \frac{\mathrm{d}p_{vs}}{\mathrm{d}T} \nabla T \right) + S_w ,$$

$$\frac{\partial H}{\partial t} = \mathrm{div}\left(h_v \delta_p p_{vs} \nabla\varphi + \left(\lambda + h_v \delta_p \varphi \frac{\mathrm{d}p_{vs}}{\mathrm{d}T} \right) \nabla T \right) .$$

More details can be found in Künzel (1995) and Černý and Rovnaníková (2002). Spatial discretization by the finite element method results in the system of ordinary differential equations which can be written

$$\begin{pmatrix} \boldsymbol{K}_{\varphi\varphi} & \boldsymbol{K}_{\varphi T} \\ \boldsymbol{K}_{T\varphi} & \boldsymbol{K}_{TT} \end{pmatrix} \begin{pmatrix} \boldsymbol{d}_\varphi \\ \boldsymbol{d}_T \end{pmatrix} + \begin{pmatrix} \boldsymbol{C}_{\varphi\varphi} & \boldsymbol{C}_{\varphi T} \\ \boldsymbol{C}_{T\varphi} & \boldsymbol{C}_{TT} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{d}}_\varphi \\ \dot{\boldsymbol{d}}_T \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}_\varphi \\ \boldsymbol{f}_T \end{pmatrix} . \tag{1}$$

Temporal discretization by the generalized trapezoidal rule transforms the previous system into the system of nonlinear algebraic equations in the form

$$(\boldsymbol{C}_n + \Delta t \gamma \boldsymbol{K}_n)\, \boldsymbol{v}_{n+1} = \boldsymbol{f}_{n+1} - \boldsymbol{K}_n \left(\boldsymbol{d}_n + \Delta t (1-\gamma) \boldsymbol{v}_n \right) , \tag{2}$$

Details can be found in Hughes (1987), Bittnar and Šejnoha (1996), Kruis and Koudelka and Krejčí (2010) and Kruis and Koudelka and Krejčí (2012).

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The conductivity matrix of material based on the Künzel assumption is populated by entries with very different order of magnitude. The better moisture insulation, the larger difference in orders of magnitude. Theoretically, a perfect hydrophobic material leads to zero term $d_{\varphi\varphi} = D_\varphi + \delta_p p_{vs}$ which results in a zero row and column in the conductivity matrix of a finite element $\boldsymbol{K}$. Moreover, there could be a zero column and row in the matrix of the whole problem which make difficulties for solvers of linear algebraic systems of equations. In the case of real materials, the diagonal term $D_\varphi + \delta_p p_{vs}$ is not exactly equal to zero but it could be very small and rows and columns in the global matrix could be nearly zero. The condition number of the global matrix is very large in such cases. It causes severe problems to iterative solvers because the rate of convergence usually depends on the condition number. If a direct solver is used for such systems of equations, significant cancellation errors could occur.

If materials with extremely small moisture conductivities are used, numerical difficulties may occur when significant moisture fluxes are presents. Such situation emerges e.g. near boundary where moisture flux is defined by external conditions. If an insulation material is close to the structure surface, the model is unable to transport the moisture flux from the exterior into structure. It results in non-balanced fluxes and the non-linear solver tends to reduce the length of time step. When the time step length is smaller than reasonable threshold, e.g. 10^{-3} s, the solver announces problems and it stops, see references Kočí et al (2012) and Kočí et al (2010).

3. Conclusions

Modification of the algorithm for solution of coupled heat and moisture transfer based on the Künzel model was introduced. It evaluates contributions to the moisture and heat fluxes and it adaptively deals with the degrees of freedom defined in nodes of finite element mesh. If some fluxes are smaller than the others, the appropriate degrees of freedom are removed from the system and perfect insulation is obtained. When material parameters change their values, the degrees of freedom are returned to the system.

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