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COMPUTATIONALLY EFFICIENT ALGORITHMS FOR EVALUATION OF STATISTICAL DESCRIPTORS

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Abstract: *Homogenization methods are becoming the most popular approach to modelling of heterogeneous materials. The main principle is to represent the heterogeneous microstructure with an equivalent homogeneous material. When dealing with the complex random microstructures, the unit cell representing exactly periodic morphology needs to be replaced by a statistically equivalent periodic unit cell (SEPUC) preserving the important material properties in the statistical manner. One of the statistical descriptors suitable for SEPUC definition is the lineal path function. It is a low-order descriptor based on a more complex fundamental function able to capture certain information about the phase connectedness. Its main disadvantage is the computational cost. In this contribution, we present the reformulation of the sequential C code for evaluation of the lineal path function into the parallel C code with Compute Unified Device Architecture (CUDA) extensions enabling the usage of computational potential of the NVIDIA graphics processing unit (GPU).*

Keywords: ***Lineal path function, homogenization, statistically equivalent periodic unit cell, graphics processing unit.***

Modelling of random heterogeneous materials is a multi-disciplinary problem with a wide range of relevant engineering applications. The unifying theoretical framework is provided by homogenization theories, which aim at the replacement of the heterogeneous microstructure with an equivalent homogeneous material, e.g. Torquato (2002). Currently, two main approaches are available: (i) computational homogenization and (ii) effective media theories. While the first class of methods studies the distribution of local fields within a typical heterogeneity pattern using a numerical method, the second group estimates the response analytically on the basis of limited geometrical information (e.g. the volume fractions of constituents) of the analysed medium.

It is generally accepted that detailed discretisation techniques, and the Finite Element Method (FEM) in particular, remain the most powerful and flexible tools available. Despite of the tedious computation time, it provides us details of local stress and strain fields. Moreover, it is convenient to characterize the material heterogeneity by introducing the concept of a Periodic Unit Cell (PUC) (Vorel, 2009) or Statistically Equivalent Periodic Unit Cell (SEPUC), see Zeman and Šejnoha (2007); Vorel et al. (2012) for more details. On the other hand, if only the overall (macroscopic) response is demanded variable, it is sufficient to introduce structural imperfections in a cumulative sense using one of the averaging schemes, e.g. the Mori-Tanaka method (Vorel and Šejnoha, 2009). If the effective material parameters of complex microstructure (see Figure 1) are demanded, the homogenization technique based on the SEPUC can be utilized. Furthermore, this approach allows us to reduce the computation cost by generating smaller unit cell describing the real structure. The generation of the SEPUC is based on optimization of an appropriate statistical descriptor. One most commonly used group of descriptors embodies a set of general n-point probability functions, applicable to an arbitrary two-phase composite. A different statistical function deserves attention when phase connectivity information is to be captured in more detail. Therefore we focus here on usage of the lineal path function. The principal drawback concerns its evaluation, which is

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non-negligible time-consuming, especially when evaluated many times within the optimization process. Hence, we present an accelerated implementation of the lineal path function on the GPU.

We have compared the sequential variant of lineal path function calculation on a single CPU with the parallel one using the GPU. The particular computations were performed on INTEL Core i7 CPU 950 @ 3.07 GHz, 12 GB RAM, GPU - NVIDIA QUADRO 4000 with Microsoft Windows Enterprise SP 1 operating system and the CUDA v. 4.0 compute capability. The efficiency of GPU parallelism was demonstrated on evaluation of lineal path function for 10 two-dimensional images with the size varying from 50x50 px to 500x500 px. One can see that for very small images, the usage of CPU outperforms the GPU because of additional time spent by copying the data from main memory RAM to GPU memory. Nevertheless, the parallelism of GPU gains for images larger than 50x50 px and the time savings increase rapidly.

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