

# APPLICATION OF DYNAMIC RELAXATION METHOD IN ANALYSIS OF CABLE MEMBRANE STRUCTURES

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**Abstract:** The article is focused on analysis of the cable membrane structure mainly the dynamic relaxation method and parameters which influence the stability and the speed of computation.

Keywords: Dynamic relaxation method, cable membrane structures, iteration parameters.

## 1. Motivation

Light cable membrane construction founds its utilization mainly on the structures where it is necessary to cover large areas like warehouses, exhibition areas and stadiums. Thanks to the modern design, many cable membrane structures were built in the last twenty years. They can be prefabricated which leads to effective build considering usage of material. One of the most noticeable aspects is design. In case of highly visible design they are significant architectonical elements.

## 2. Design of cable membrane structures

By Topping and Iványi (2007) there are several steps which are necessary for design cable membrane structure. After the shape definition the engineering model can be created and the main parameters for *form finding* are defined. During the form finding process the equilibrium state of cable membrane structure is found and the specific boundary conditions are obtained from this process. The final equilibrium state is found by optimizations methods. In the next step, the response of construction to loading is *analysed*. Now, the appropriate shape of structure is known and it is necessary to make the *cutting pattern generation* and *design the details*. This minimizes the wastage of material during the production plan parts of construction from roll of material.

## 3. Cable membrane structures analysis

Various methods are used for cable membrane structure analysis. One of the simplest methods is *grid method*. Further, there are simply numerical methods, *finite difference method* and *finite element method*. Nowadays it is possible to solve constructions with arbitrary irregular shape and prestressed construction using these methods.

From many various methods there are two others suitable. *Force density method* which is based on the constant ratio between the force in the element and the length of the element. At last the *dynamic relaxation method* which is highly used for the form finding and analysis of construction.

## 4. Dynamic relaxation

Dynamic relaxation is not used for finding dynamic response of construction but it is used for static problems using a fictitious dynamic analysis. In this method the motion of construction from the time

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of loading to the state of equilibrium is traced step by step. From the motion it is possible to determine the curve of the construction without compile the matrix of stiffness.

The method is a direct application of Newton's second law of motion. During the static analysis of construction the fictitious damping is used. Iteration to the static solution is relatively fast when critically damped or overdamped construction is used.

### 4.1. Numerical procedure

The Newton's second law of motion is presented in the equation which describes residual forces in time *t* and joint *i*. To the calculation of residual forces it is necessary to add the effect of prestress. The Equation (1) describes the calculation of residual forces in *x* direction with influence of the average velocity and the acceleration over the time step  $\Delta t$ :

$$R_{ix}^{t} = \frac{M_{ix}}{\Delta t} \cdot \left( v_{ix}^{(t+\Delta t/2)} - v_{ix}^{(t-\Delta t/2)} \right) + \frac{C_{ix}}{2} \cdot \left( v_{ix}^{(t+\Delta t/2)} - v_{ix}^{(t-\Delta t/2)} \right)$$
(1)

where:

 $R_{ix}^{t}$  is the residual force at joint *i* at time *t* 

 $M_{ix}$  is the fictitious mass at joint *i* 

 $C_{ix}$  is the viscous damping factor for joint *i* 

 $\dot{v}_{ix}^{t}, v_{ix}^{t}$  are the acceleration and velocity at the time t at joint i.

Rearrangement of Equation (1) enables to calculate the velocity at the new time step  $(t+\Delta t)$ . In the next step the current coordinates of joint *i* are calculated. For calculating the current coordinates of joint *i* at the end of the first time step  $(x_i^{(t+\Delta t)})$  it is necessary to set the initial conditions for time t=0.

### 4.2. Dynamic relaxation method stability and convergence

The stability and convergence of the dynamic relaxation is influenced by the distribution of fictitious nodal mass, the damping factor and the time interval of the step. During the calculation fixed time step is often used and other factors are tuned until the required accuracy and stability of calculation is reached. When the time step  $\Delta t$  exceeds a critical value or fictitious masses are too low, numerical instability of the calculations will occur and the equilibrium state cannot be reached. This shortage can be eliminated by decreasing the time step or increasing the fictitious masses.

From the previously stated it is obvious that the tuning of the calculation parameters (time step, fictitious masses and damping factor) is really an attractive area of interest since these parameters have a large influence on the speed of calculation. Beside these factors are specific for each construction. In full text some calculated examples are shown the same as main advantages and drawbacks are mentioned.

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