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MIXED-MODE HIGHER-ORDER TERMS COEFFICIENTS ESTIMATED USING THE OVER-DETERMINISTIC METHOD

L. Šestáková*

Abstract: *The so-called 'over-deterministic' method (ODM) is applied on a mixed-mode configuration in order to determine higher-order terms coefficients of the Williams expansion approximating the stress and displacement fields in a cracked body. Results obtained agree very well to data found in literature (calculated by means of hybrid crack elements and boundary collocation method) and therefore further convergence studies are made in order to find some restrictions and recommendations corresponding to use of the ODM. Note, that more than five terms of the Williams expansions are considered and investigated in this contribution.*

Keywords: ***Crack, Williams expansion, higher-order terms coefficients, FE analysis, over-deterministic method.***

1. Introduction

It has been shown in recent years that conventional linear elastic fracture mechanics is not suitable in the case of quasi-brittle materials. Therefore, the so-called over-deterministic method (ODM) has been used in order to calculate the mixed-mode higher-order terms of the Williams expansion (Williams, 1957) that are of great relevance because they can predict the constraint of crack tip fields (Berto & Lazzarin, 2010) and interpret the size/geometry/boundary effect (Karihaloo et al., 2003).

2. Problem description

A mixed-mode cracked specimen configuration has been chosen for testing of the ODM in this paper. Higher-order terms have been estimated on a plate with an angled edge-crack under uniaxial tension (AECT), whereas the crack angle was chosen as $\beta = 30°$, see (Ayatollahi & Nejati, 2010) for detailed geometry. In order to obtain the displacement field near the crack tip, a numerical model was created.

3. Results and discussion

An elementary goal of this work was to validate the ODM procedure on a mixed-mode configuration in order to obtain a reliable tool for further analysis of the stress field in quasi-brittle materials. This was successfully done, see Tab. 1

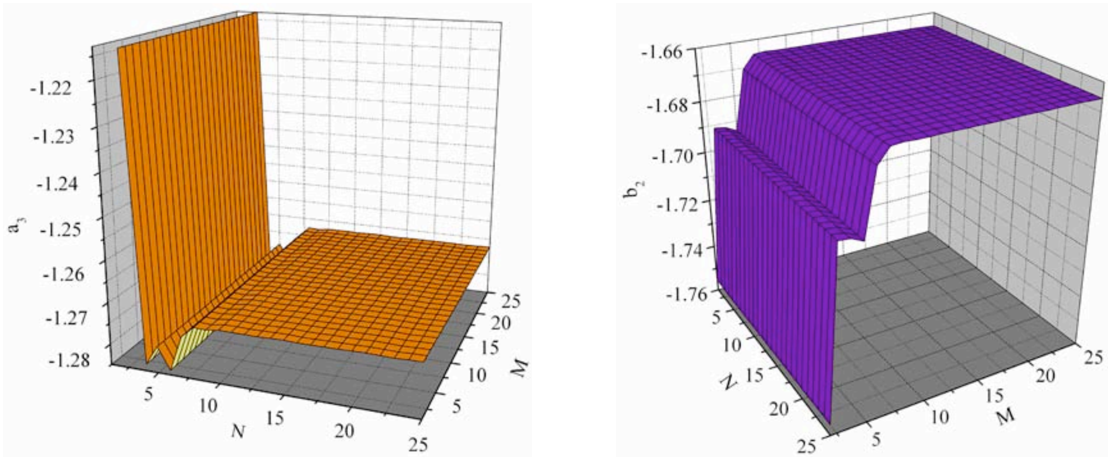
Tab. 1: Higher-order terms coefficients determined by means of the ODM (3. column) in comparison to data published in literature (Xiao et al., 2004)

	HCE	BCM	ODM
a_1	1.3867	1.3918	1.3938
a_2	0.1463	0.1485	0.1493
a_3	-1.2416	-1.2681	-1.2724
a_4	0.2400	0.2468	0.2472
a_5	-0.5226	-0.5422	-0.5433

	HCE	BCM	ODM
b_1	-0.3762	-0.3777	-0.3784
b_2			-1.6653
b_3	-0.2141	-0.2213	-0.2210
b_4	-0.1954	-0.1906	-0.1928
b_5	0.1888	0.1792	0.1807

* Ing. Lucie Šestáková, Ph.D.: Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology; Veveří 331/95; 60200, Brno; CZ, e-mail: sestakova.l@fce.vutbr.cz

Several convergence studies have been done, see for instance the higher-order terms convergence with increasing number of the Williams series expansion terms N and M in Fig. 1.



a) b)

Fig. 3: Higher-order terms a_3 and b_2 convergence in dependence on the number of Williams expansion coefficients N, M considered during calculations.

4. Conclusion

A numerical study has been carried out in order to test and validate the so-called over-deterministic method suggested for determination of higher-order terms coefficients in Williams expansion describing the stress field near a crack tip. A very good agreement has been found between the mixed-mode coefficients calculated and data published in literature for a plate with an angled edge-crack under uniaxial tension. On the basis of a convergence study performed, a recommendation on a symmetrical mesh around the crack tip (and symmetrical displacements field, respectively) can be expressed in order to avoid the general 3-dimensional dependence of higher-order terms on the number of terms considered during calculations N, M. Further, it has been observed that only first several terms (let's say up to $N = M = 13$) converge clearly with the increasing number of terms considered during calculations; the higher ones are not sure and thus, additional studies should be done in this direction because these terms can be important for better understanding of fracture in quasi-brittle materials as well.

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References

Ayatollahi, M.R., Nejati, M. (2010) An over-deterministic method for calculation of coefficients of crack tip asymptotic field from finite element analysis. *Fatigue & Fracture of Engineering Materials & Structures*, 34(3) , pp. 159-176

Berto, F., Lazzarin, P. (2010) On higher order terms in the crack tip stress field. *International Journal of Fracture*, 161(2), pp. 221-226.

Karihaloo, B.L., Abdalla, H.M., Xiao, Q.Z. (2003) Size effect in concrete beams. *Engineering Fracture Mechanics*, 70, pp. 979-993.

Williams, M.L. (1957) On the stress distribution at the base of a stationary crack. *Journal of Applied Mechanics*, 24, pp.109-114.

Xiao, Q.Z., Karihaloo, B.L., Liu, X.Y. (2004) Direct determination of SIF and higher order terms of mixed mode cracks by a hybrid crack element. *International Journal of Fracture*, 125, pp. 207-225