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Bending, Torsion and Distortion of Thin-Walled Beams

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Abstract: Analysis of thin-walled beams using modified Generalised Beam Theory (GBT) developed by R. Schardt [1, 2] for calculation of cross-section properties. Numerical examples show in details the calculation of internal forces, deformations and stresses due to torsion and distortion with the help of: (i) exact formulae, (ii) computer program IQ 100 using analogies with beams in bending on elastic foundations loaded by transverse load and tension force. Formulae for open and closed profiles. Influence of shear is analysed too [5].

Introduction

The theory of thin-walled structures was developed by Vasilij Zacharovič Vlasov, who published basic publications in 1931, 1935, 1936, 1940 and seven monographs in 1933, 1936, 1940, 1949, 1949 and 1958. Generalized Beam Theory (GBT), which describes the mechanical behaviour of prismatic structures in the 1st order theory by ordinary differential equations

$$E.{}^{k}C.{}^{k}V^{IV}\left(x\right)-G.{}^{k}D.{}^{k}V^{II}\left(x\right)+{}^{k}B.{}^{k}V\left(x\right)={}^{k}q\left(x\right) \qquad (1)$$

for each mode k, which are not coupled and can be solved independently, was developed in Germany by Richard Schardt in 1966 [1] and by Gerhard Sedlacek in 1968 [3]. G. Sedlacek gave to this theory its name. R. Schardt published the bible of GBT in 1989 [2]. Details and list of all references may be found in [4, 5]. Eq. 1 does not take into account influence of shear. Influence of shear is investigated in numerical examples.

Cross-sectional functions, stiffnesses, internal forces, deformations and stresses

A unifying feature of the GBT is the concept of warping functions whereby each mode k of deformation is associated with a distribution of axial strain ${}^{k}u(s)$. The first mode (k = 1) of deformation is a uniform distribution of axial strain over cross-section. The second and third modes (k = 2, 3) are bending modes and the associated warping functions are linear distributions strain about the two principal axes. The fourth mode (k = 4) is torsion and here the term warping has its conventional meaning as the warping function is the sectorial coordinate which reflects the distribution of axial strain due to a bimoment. The fifth (k = 5) and further modes (k > 5) are the modes of cross-sectional distortions. It is important to realize at the outset that all these warping functions are orthogonal. Practically, this means that in any first-order analysis they can be considered quite independently and their effects combined by simple superposition. The cross-sectional functions of C-profile (k = 1 to 6) are calculated in numerical example and illustrated graphically in tables. The C-profile has 5 plane elements (n = 5) and 6 main nodes (n + 1 = 6). Nodes numbering is r = 0; 1; 2; 3; 4; 5. It should be noted that all the quantities tabulated for modes k = 1 to k = 4 are already obtainable from standard procedures of structural mechanics which are given in basic texts on the subject. The details of the calculation of the warping functions ${}^{k}u(s)$ [or $-{}^{k}u(s)=1$, $y(s)$, $z(s)$, $\omega(s)$, $\omega_{D1}(s)$, $\omega_{D2}(s)$] may be found also in [4, 5].

The values ${}^{k}Q_{r}$ may be calculated from the first moments of area using the following formulae

$$ {}^{k}Q_{i\text{-}j} = \left({}^{k}S_{i} + {}^{k}S_{j} + \frac{\left({}^{k}u_{j} - {}^{k}u_{i}\right) t_{i\text{-}j} L_{i\text{-}j}}{6} \right) \frac{L_{i\text{-}j}}{2} = \left({}^{k}S_{i} + {}^{k}S_{j} + \frac{{}^{k}f_{s,i\text{-}j}\, t_{i\text{-}j} L_{i\text{-}j}^{2}}{6} \right) \frac{L_{i\text{-}j}}{2} \tag{2} $$

The ${}^{k}C^{M}$ warping constants, which include also the cross-sectional area ${}^{1}C^{M} = A$, the second moments of area about the principal axes ${}^{2}C^{M} = I_{z}$, ${}^{2}C^{M} = I_{y}$ and the warping constant ${}^{4}C^{M} = I_{w}$, may be calculated from the following formulae

$$ {}^{k}C^{M} = \int_{0}^{n} \left[{}^{k}u(s) \right]^{2} ds = \int_{0}^{n} -\frac{d\left[{}^{k}u(s)\right]}{ds} {}^{k}S(s)\, ds = \sum_{r=1}^{n=5} -\frac{d\left[{}^{k}u(s)\right]}{ds} {}^{k}Q_{r} = \sum_{r=1}^{n=5} -{}^{k}f_{s,r}\, {}^{k}Q_{r} \tag{3} $$

The “torsion” constants may be calculated from the formulae

$$ {}^{k}D = \frac{1}{3} \sum_{r=1}^{n=5} {}^{k}f_{v,r}^{2} L_{r} t_{r}^{3} \quad \text{for} \quad k = 4,5,6, \qquad {}^{k}D = 0 \ \text{(it does not exist)} \ \text{for} \quad k = 1,2,3 \tag{4} $$

The transverse bending stiffnesses ${}^{k}B$ may be calculated from the formulae

$$ {}^{k}B = \sum_{i=0}^{n=5} {}^{k}m_{s,i}\, {}^{k}V_{i} \quad \text{for} \quad k = 5,6, \qquad {}^{k}B = 0 \ \text{(it does not exist)} \ \text{for} \quad k = 1,2,3,4 \tag{5} $$

Summary

Numerical example shows in details the calculation of all cross-sectional functions and stiffnesses. Internal forces and deformations of torsion and distortion are obtained by computer program enabling to calculate beam in bending on elastic foundation loaded by transverse and tension forces.

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