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On Choice of Lagrange Multipliers for Fictitious Domain Method for the Numerical Simulation of Incompressible Viscous Flow around Rigid Bodies

Matej Beňo[a], Bořek Patzák[b]

Faculty of Civil Engineering, Czech Technical University, Thákurova 7, 166 29, Prague CZ

[a]matej.beno@fsv.cvut.cz, [b]borek.patzak@fsv.cvut.cz

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Abstract: The aim of this article is to present the results of flow around rigid obstacles. The finite element implementation is based on the incompressible Navier-Stokes equation on structured, two dimensional triangular meshes using operator splitting for time discretization. To model rigid particles, the fictitious domain method is used. The article compares different approaches for selection of Lagrange multipliers used to enforce rigid body constrains.

Introduction

The fictitious domain method represents useful and powerful tool for modeling of fluid flows with obstacles. The principle of this method is to extend a problem domain to include not only the fluid, but also the particles. This lead to so called "fictitious" domain. This domain is geometrically simpler, which allows to use more regular meshes. The advantage of this approach is that specific fast solvers can be developed. The second main advantage is that there is no need to compute the hydrodynamics forces and torques.

Method

The fictitious domain method used in this article follows the work of Glowinsky [1]. The use of a combined weak formulation for fluid-structure system leads to cancelation of the forces and moments between the obstacles and fluid. Then the fluid flow is solved from weak formulation everywhere in the domain, including the fictitious domain parts inside the obstacles. To enforce the rigid-body constraints on obstacles, the Lagrange multipliers are used. The multipliers represents the additional body force needed to maintain the rigid body motion inside the obstacles.

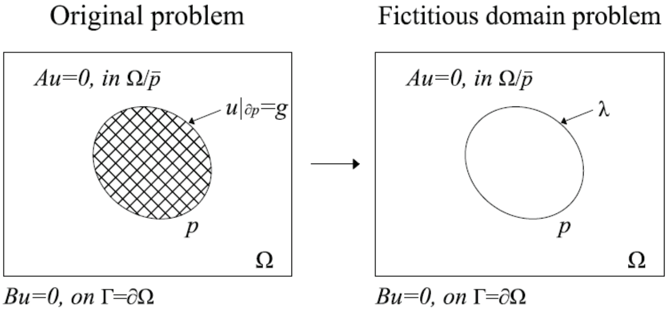

Fig. 1: Original and fictitious domain problem.

Numerical results

The numerical simulation of flow around rigid object is performed on model shown in Fig.2. The domain has been divided into 4256 elements. The perfect friction on both plates (zero velocity) has been assumed. The following parameters have been used: mass density $\rho = 1.0\, kg/m^3$, viscosity $\eta = 10^{-2} Pa\, s$, and time step $\Delta t = 0.05$. These results are presented in steady state.

To enforce no movement inside the rigid particle, the rigid movement condition is enforced using Lagrange multipliers on selected sampling points. Various strategies are tested on how to select different sampling points on the particle boundary and volume. Profiles of velocities in vertical sections at different positions are compared in Fig 3. The results are also compared using the results obtained by traditional approach, where only fluid part has been discretized.

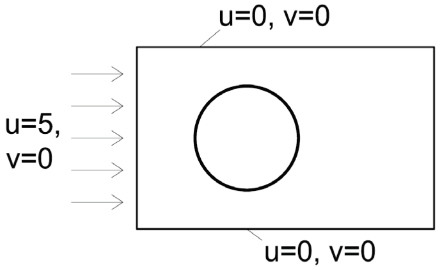

Fig. 2: The geometry and the boundary conditions of flow in tube with obstacle.

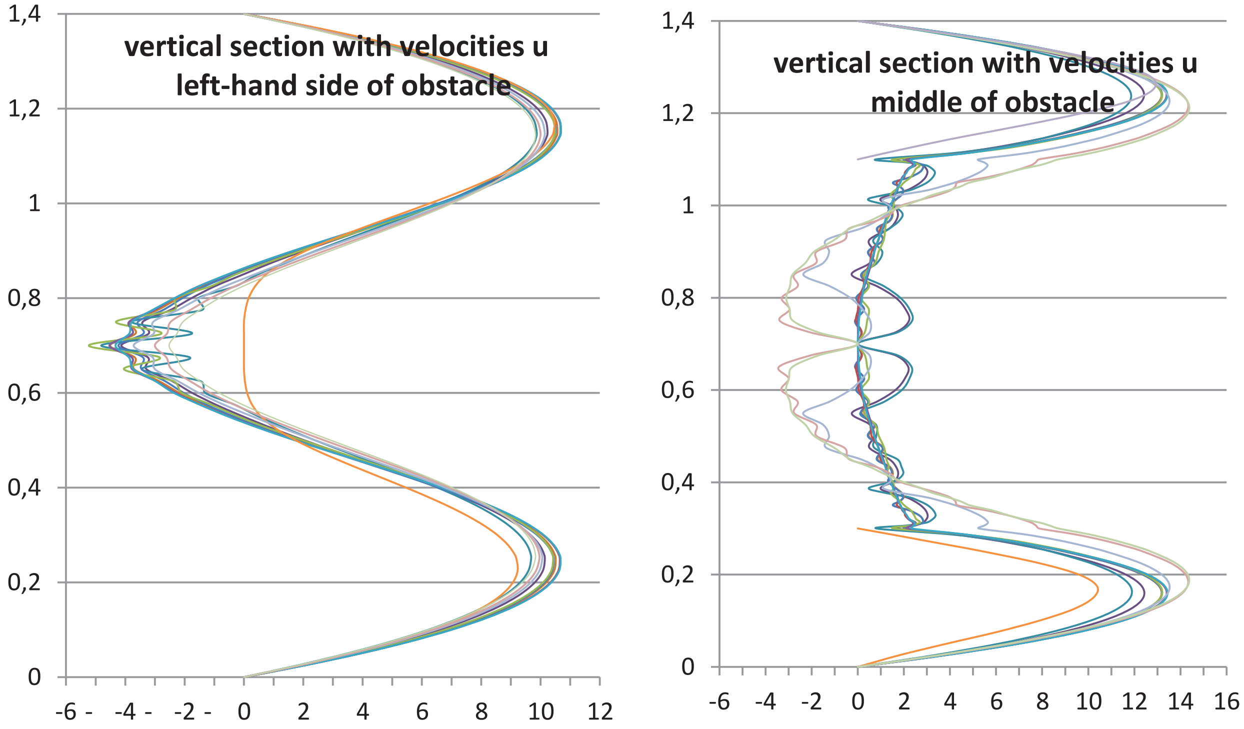

Fig. 3: Velocity profiles in front of and through obstacle.

Conclusion

Presented results show that the fictitious domain approach is capable to accurately predict the flow profiles, however, the quality strongly depends on selected strategy of sampling point selection. The optimal results have been obtained for strategy based on fifteen and more Lagrange multipliers uniformly selected throughout the obstacle.

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