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Modelling of Rotating Twisted Blades as 1D Continuum

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Abstract: Presented paper deals with modelling of a twisted blade with rhombic shroud as one-dimensional continuum by using beam finite elements with linearly varying cross-sectional parameters. The blade is clamped into a rotating rigid disk and the shroud is considered as a rigid body. Besides, in consequence of eigenfrequencies comparison with a 3D finite element model of the identical blade, material properties tuning is performed.

Introduction

In some publications dealing with modelling of turbine bladed disks, the disk is modelled by using 3D finite elements, but the blades are considered as 1D continuum modelled by using Rayleigh beam elements [1, 2]. This method can be perceived as a basic tool for investigation of dynamic behaviour of the bladed disks and an alternative to more complex 3D finite element models of the blades [3]. Due to lower number of degrees of freedom, this approach can be advantageously used, for example, for geometric properties optimization because of less time-consuming computation.

However, due to Bernoulli-Navier hypothesis considering (that says: during deflection, the plane normal cross-sections of the blade remain plane and normal to the deflected longitudinal axis of the blade), the turbine blades modelled by using 1D beam elements are stiffer than the 3D finite element models. This phenomenon is pronounced especially in case of so-called “short blades”. Therefore, material properties tuning of the 1D blade model is one of the options to attenuate this handicap.

The aim of this work is modelling of rotating twisted blade as 1D continuum using material properties tuning as a result of eigenfrequencies comparison with a 3D finite element model.

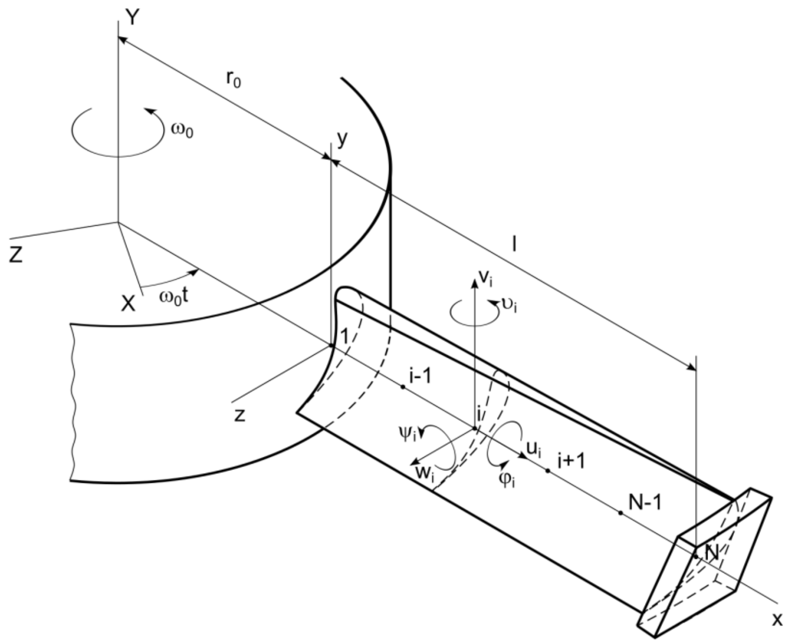

Fig. 1: Rotating twisted blade clamped into a rigid disk

Blade modelling and optimization

The computational model of a twisted blade with rhombic shroud (that is considered as rigid) and variable cross-sections along the blade is shown in Fig. 1 [4]. This blade is clamped into a rigid disk rotating with constant angular velocity ω_0. According to introduction, the blade is divided into N beam finite elements with linearly varying cross-sectional parameters and six degrees of freedom in every node [5]. Then, the equations of motion of the blade can be expressed in generalized coordinates

$$\boldsymbol{q_B} = [. . . u_i, v_i, w_i, \varphi_i, \vartheta_i, \psi_i, . . .]^T , \quad i = 1, . . ., N \tag{1}$$

in following form

$$\boldsymbol{M_B}\ddot{\boldsymbol{q}}_{\boldsymbol{B}} + (\boldsymbol{B_B} + \omega_0 \boldsymbol{G_B})\dot{\boldsymbol{q}}_{\boldsymbol{B}} + \left[\boldsymbol{K_{s,B}} + \omega_0^2(\boldsymbol{K_{stiff,B}} - \boldsymbol{K_{\omega,B}})\right]\boldsymbol{q_B} = \boldsymbol{0}, \tag{2}$$

where symmetric matrices $\boldsymbol{M_B}$, $\boldsymbol{B_B}$, $\boldsymbol{K_{s,B}}$, ${\omega_0}^2\boldsymbol{K_{stiff,B}}$, ${\omega_0}^2\boldsymbol{K_{\omega,B}}$ are mass, material damping, static stiffness, centrifugal stiffening under rotation and softening because of modelling in a rotating coordinate system, respectively. The skew symmetric matrix $\omega_0\boldsymbol{G_B}$ represents gyroscopic effects.

Due to higher stiffness as a result of Bernoulli-Navier hypothesis considering, parameters E (Young's modulus), ν (Poisson's ratio) and J_k (torsion resistance) of every beam finite element were being tuned. The objective function has the form

$$\psi(\boldsymbol{p}) = \sum_{j=1}^{m} \left[(m + 1 - j)^2 . \left(\frac{\Omega_j(\boldsymbol{p}) - \Omega_j^*}{\Omega_j^*}\right)^2\right], \tag{3}$$

where $\boldsymbol{p}$ is vector of optimization parameters, $\Omega_j(\boldsymbol{p})$ are eigenfrequencies of the 1D finite element model and Ω_j^* are eigenfrequencies of the 3D finite element model of the identical blade.

Summary

Modelling of turbine twisted blades by using 1D beam finite elements with variable cross-sectional parameters can be an alternative to large 3D finite element models, especially in cases of parameter optimization or contact problems of bladed disks. However, the blades modelled by this method based on Bernoulli-Navier hypothesis are stiffer. Parameter tuning is a suitable tool to attenuate this handicap.

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References

[1] J. Šašek, V. Zeman, J. Kellner, Modal Condensation of Bladed Disk, Engineering Mechanics, 19 (2012) 351.

[2] J. Kellner, Vibrations of Turbine Blades and Bladed Disks, PhD Thesis, University of West Bohemia, Pilsen, 2009 (in Czech).

[3] M. Hajžman, M. Byrtus, V. Zeman, Solution of the Mutual Contact in the Finite Element Analysis of Twisted Blades, Colloquium Dynamics of Machines 2012, Prague, pp. 51-58.

[4] Z. Kubín, J. Hlous, Measurement of Rhombic Bladed Disk, Research report, Doosan Škoda Power, 2012.

[5] M. Byrtus, M. Hajžman, V. Zeman, Dynamics of Rotating Systems, University of West Bohemia, Pilsen, 2010 (in Czech).