21st International Conference **ENGINEERING MECHANICS 2015**
Svratka, Czech Republic, May 11 –14, 2015
Paper **#049**, **pp. 56–57**

Bifurcations in Mathematical Model of Nonlinear Vibration of the Nuclear Fuel Rod

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Keywords: bifurcation, vibration, nonlinear dynamics, nuclear fuel rods

Abstract: The paper deals with nonlinear phenomena occurring during vibration of nuclear fuel rod (FR). The FR is considered as a system consisting of two impact-interacting subsystems - FR cladding (zircalloy tube) and fuel pellets stack placed inside FR cladding. Between both subsystems, there is a small clearance. The FR is bottom-end fixed, and at eight equidistant levels, the FR cladding is supported by spacer grids (SG). Both subsystems are modelled by means of finite element method for one-dimensional Euler-Bernoulli continua. During fuel assembly (FA) motion caused by pressure pulsations of the coolant, the FR vibrates and impacts can possibly occur between FR cladding and fuel pellets stack. The paper focuses on qualitative change of vibration with change of bifurcation parameters - clearance between FR cladding and fuel pellets stack, stiffness of spacer grids cells and excitation frequency. The change of vibration quality is shown using extremes of relative radial displacements of both tubes in discretization nodes, and phase trajectories.

Introduction

Nuclear fuel rods (FR) are key part of the nuclear power plants (NPP). FRs are arranged into fuel assemblies (FA), which are submerged into coolant and fit into lower piece. Because of coolant pressure pulsations caused by main circulation pumps, vibration of the lower piece is induced and that is the cause of FA (and every FR included) kinematical excitation. Every FR consists of two parts - long thin-walled zircalloy tube (C) and UO_2 fuel pellets stack (P) fit inside (see Fig. 1). Between these two subsystems, there is a clearance which varies during reactor operational cycle. The FR cladding is supported by spacer grid cells at eight levels. The concept of FA components modelling for hexagonal type FA is borrowed from [1].

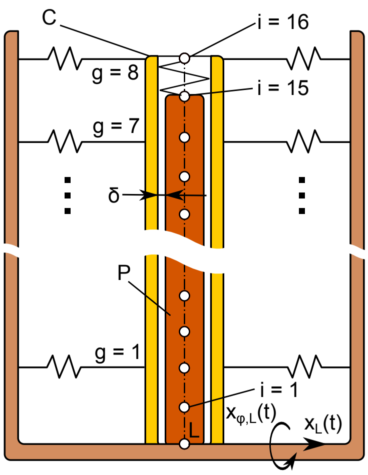

Fig. 1: Schematic diagram of the FR consisting of fuel pellets stack (P) and FR cladding (C) coupled by spacer grids cells to rigid skeleton

Mathematical model and bifurcations

Both subsystems P and C were modelled using finite element method (FEM) for Euler-Bernoulli continua [2] and only flexural vibration was considered. Lower nodes L of both subsystems are fixed into lower piece whose motion is supposed to be harmonic. FA skeleton is considered as rigid and its motion is given by L point motion. Subsystem P is divided into 15 elements and subsystem C into 16 elements. Upper nodes of both subsystems are coupled by fixation spring. Denoting vector $\boldsymbol{q}_F^{(Y)}$ of generalized coordinates of the free nodes of the subsystem $Y = P, C$, the mathematical model of the system consisting of two subsystems can be written in the form

$$\begin{bmatrix} \boldsymbol{M}_F^{(C)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_F^{(P)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_F^{(C)} \\ \ddot{\boldsymbol{q}}_F^{(P)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_F^{(C)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_F^{(P)} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{q}}_F^{(C)} \\ \dot{\boldsymbol{q}}_F^{(P)} \end{bmatrix} + \left(\begin{bmatrix} \boldsymbol{K}_F^{(C)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_F^{(P)} \end{bmatrix} + \boldsymbol{K}_{fix} \right) \begin{bmatrix} \boldsymbol{q}_F^{(C)} \\ \boldsymbol{q}_F^{(P)} \end{bmatrix} =$$

$$= \begin{bmatrix} \boldsymbol{f}_L^{(C)}(t) \\ \boldsymbol{f}_L^{(P)}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{SG,C}(t) \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{f}_{P,C}(\boldsymbol{q}_F^{(C)}, \boldsymbol{q}_F^{(P)}) \\ \boldsymbol{f}_{C,P}(\boldsymbol{q}_F^{(C)}, \boldsymbol{q}_F^{(P)}) \end{bmatrix}, \quad (1)$$

where $\boldsymbol{X}_F^{(Y)}, \boldsymbol{X} = \boldsymbol{M}, \boldsymbol{B}, \boldsymbol{K}$ are mass, damping and stiffness matrices corresponding to free nodes of subsystems $Y = P, C$, matrix $\boldsymbol{K}_{fix}$ expresses coupling of fixation spring, vectors $\boldsymbol{f}_{P,C}(\boldsymbol{q}_F^{(C)}, \boldsymbol{q}_F^{(P)}), \boldsymbol{f}_{C,P}(\boldsymbol{q}_F^{(C)}, \boldsymbol{q}_F^{(P)})$ represent impact forces between P and C, vector $\boldsymbol{f}_{SG,C}(t)$ represents excitation caused by motion of the skeleton and $\boldsymbol{f}_L^{(C)}, \boldsymbol{f}_L^{(P)}$ represent kinematical excitation of lower nodes of both subsystems.

Time solution of the system described by Eq. 1 is obtained performing numerical integration in time domain using fourth order Runge-Kutta method. As bifurcation parameters, the clearance δ, spacer grid cells stiffness and excitation frequency were chosen. The qualitative change of vibra-

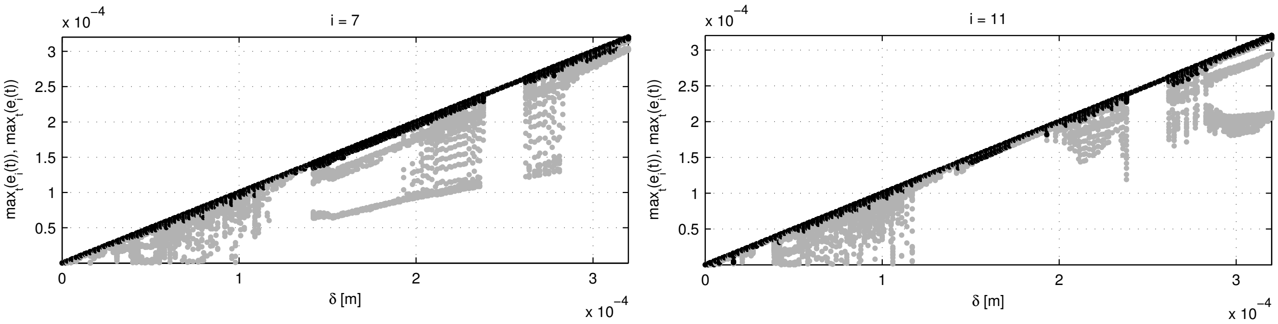

Fig. 2: Bifurcation diagram for maxima and minima of relative displacements $e_i(t)$ in node $i = 7, 11$ dependent on radial clearance between P and C

tion is studied using bifurcation diagrams for extremes $\hat{e}_i$ of relative radial displacements $e_i(t)$ of both subsystems in nodes $i = 1, \ldots 15$. As an example, Fig. 2 shows bifurcation diagram for clearance δ in nodes $i = 7, 11$. Equivalent diagrams were constructed for stiffnesses $k_g, g = 1, \ldots 8$ of spacer grids cells and excitation frequency ω as bifurcation parameters.

Conclusion

As described above, simplified mathematical model of a FR was formulated to describe quantitative change of FR vibration. The change is studied using bifurcation diagrams which enables to distinguish between bands with smooth operation and bands with chaotic motion which is often characterized by possible undesirable phenomena such as noise emission and wear.

Acknowledgement: This work was supported by the European Regional Development Fund (ERDF), project "NTIS" - New Technologies for the Information Society", European Centre of Excellence, CZ.1.05/1.1.00/02.0090 and the grant SGS-2013-036.

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