21st International Conference **ENGINEERING MECHANICS 2015**
Svratka, Czech Republic, May 11 –14, 2015
Paper **#221, pp. 60–61**

Discontinuous Galerkin Simulation of Dynamic Elasticity and Application to Fluid-Structure Interaction

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Keywords: dynamic linear and nonlinear elasticity, St. Venant-Kirchhoff model, compressible flow, ALE method, discontinuous Galerkin method

Abstract: This paper is concerned with the numerical simulation of the interaction of compressible viscous flow with a nonlinear elastic structure. The flow is described by the compressible Navier-Stokes equations written in the arbitrary Lagrangian-Eulerian (ALE) form. For the elastic deformation linear model or nonlinear St. Venant-Kirchhoff model is used. In the discretization the discontinuous Galerkin (DG) finite element method is applied both for the flow problem in a time-dependent domain and for the dynamic elasticity system. We show that the DG method is applicable to the discretization of both problems.

Introduction

Fluid-structure interaction (FSI) problems are solved in various aerospace, civil and mechanical engineering applications. Recently the methods for solution of such problems have also been quickly developed in the field of biomechanics. We are particularly interested in biomechanics of voice, where the methods of numerical simulation of human vocal folds self-oscillation are currently in an intensive development. These self-oscillations, which originate in the interaction of airflow coming from the human lungs with the compliant biological tissue of the vocal folds, produce primary sound enabling voicing (phonation, speech, singing). The numerical simulation of this problem requires the solution of dynamic elasticity equations coupled with the solution of the compressible Navier-Stokes equations in a time-dependent domain. Here we apply the DG method to the solution of both problems.

DG discretization of dynamic elasticity problem and the compressible Navier-Stokes equations

The numerical simulation of fluid-structure interaction requires the solution of dynamic elasticity equations coupled with the solution of the compressible Navier-Stokes equations in a time-dependent domain. It appears that a sufficiently robust and accurate method for the solution of both elasticity and flow problems is the discontinuous Galerkin method using piecewise polynomial approximations over a finite element mesh without any requirement on the continuity of approximate solutions.

In [3] and [4], the dynamic linear elasticity initial-boundary value problem is discretized by the conforming finite element method in space and by the Newmark method in time. In these papers this method was coupled with the DG technique for the simulation of compressible flow. It is logical to ask, if it would be suitable to use DG techniques for the solution of flow as well as elasticity problems. Here we shall pay attention to the solution of the dynamic elasticity problem by the DG method applied to the space discretization. The discretization in time is carried out by the backward Euler (BE) difference or general backward difference formula (BDF) or the DG method in time. We are concerned with linear elasticity model as well as nonlinear St. Venant-Kirchhoff model. In this case the nonlinear discrete problem is solved by the Nawton-Raphson method. Numerical experiments show that as a most accurate technique the space-time DG method appears and gives equally accurate results in the case of the benchmark as in [6]. For details we can refer to works [2] (linear dynamic elasticity) and [5] (nonlinear dynamic elasticity).

The compressible Navier-Stokes system in a time-dependent domain is written in the ALE form and discretized by semi-implicit space-time DG method. This method is very efficient, because on each time level it is necessary to solve only one linear algebraic system. For details see [1].

The coupled FSI problem is solved by the so-called staggered approach, which means that both problems are solved at a given time instant separately. The approximate solutions are required to fulfill certain transient conditions, which are met through several inner iterations. The DG method can also be used for the construction of the ALE mapping.

Summary

In this paper the DG discretization of a linear elasticity and nonlinear elasticity problem with St. Venant-Kirchhoff material was presented. We paid a special attention to the comparison of several different techniques for the time discretization. The space-time discontinuous Galerkin method showed promising results, in particular the ability to use a larger time step without suffering from the numerical dissipation. The results of numerical experiments demonstrate that the DG method can successfully be applied to all problems involved in FSI: fluid flow, large elastic material deformation and the construction of ALE mapping.

Acknowledgments: This research was financially supported by the projects 13-00522S (M. Feistauer) and P101/11/0207 (J. Horáček) of the Czech Science Foundation and by the grant SVV-2015-260226 of the Charles University in Prague (M. Hadrava and A. Kosík).

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