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Response of a Non-Linearly Damped Duffing Oscillator Including Non-Linear Restoring Force by Using a Variational Approach

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Abstract: In this paper, we proposed to use approximation methods for solving undamped nonlinear Duffing's equation including nonlinear restoring forces. The purpose is the application of variational approach and parameterized perturbation method to gain the natural frequency and response of the system in a class of Duffing's oscillators. It is shown the variational approach is an explicit method with high validity for the solution of strongly nonlinear oscillatory systems.

Introduction

Nonlinear oscillatory systems are important in engineering because many practical engineering components are modeled using this systems such as elastic beams supported by two springs, mass-on-moving-belt, nonlinear oscillations of a pendulum and vibration of a milling machine. Various kinds of analytical and numerical methods were used to handle the problem and other nonlinear problems such as Non–perturbation methods, variational iteration method, homotopy perturbation method, perturbation techniques, energy balance method, Lindstedt–Poincaré method, parameter–expansion method, variational approach, and Parameterized perturbation method. In this paper, we apply the variational approach and Parameterized perturbation method to the undamped nonlinear Duffing's oscillator.

Description of the Duffing's Oscillator with nonlinear restoring forces

The differential equation:

$$\ddot{x}(t) + x(t) + \varepsilon\,\alpha\,x(t)^3 = 0, \qquad \varepsilon > 0 \tag{1}$$

Is called the Duffing's oscillator which x and t are generalized dimensionless displacement and time variables, respectively, and α and ε are constant parameters in nonlinear Duffing's oscillator. It can be the model of a structural system which includes nonlinear restoring forces (for example springs).

The exact frequency for a dynamic system governed by Eq. 1 can be derived as in Eq. 2:

$$\omega_e(A) = 2\pi\left(4\int_0^{\pi/2}\frac{dt}{\sqrt{1-\frac{\alpha\varepsilon}{2}A^2(1+\cos^2 t)}}\right)^{-1} \tag{2}$$

Solution using Parameterized perturbation method

The procedure to find the solution of Eq. 1 by means of parameterized perturbation technique is given elsewhere. The closed form solution is:

$$x(t) = A\cos\omega t + \left(\frac{A^3\alpha\,\varepsilon}{32\omega^2}\right)\left(\cos(3\omega t) - \cos(\omega t)\right) \tag{3}$$

where:

$$\omega = \frac{1}{2}\sqrt{4 + 3\alpha\varepsilon A^2} \tag{4}$$

Solution procedures for Duffing equation using variational approach

This Solution given in reference [8]

$$x(t) = A\cos\left(\frac{1}{2}\sqrt{4+3\alpha\varepsilon A^2}\, t\right) \tag{5}$$

Table 1: Comparison of angular frequencies in Eq. 1 from various approximations of the Variational approach and the Parameterized perturbation method with the exact solution

Constants			Results			
A	α	ε	Exact solution ω_e	Variational approach ω	Parameterized perturbation ω	percentage error
1	-1/6	0.5	0.94119	0.96825	0.96825	2.80 %
1	-1/6	1	0.88889	0.93542	0.93542	4.98 %
1	-1/6	3	0.72727	0.79058	0.79058	8.00 %
1	-1/6	5	0.66667	0.70710	0.70710	5.72%

Summary

Nonlinear Duffing’s oscillator is sometimes used as an approximation for the pendulum. Approximate analytical approaches have been developed for solving the undamped nonlinear Duffing’s oscillator including nonlinear restoring forces. These approaches can be easily extended and applied to other nonlinear oscillation problems in engineering and science.

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