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Numerical Solution of Humid Air Flow with Non-Equilibrium Condensation

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Abstract: Fast expansion of humid air (e.g. flow around an aircraft or a flow in the test section of a wind tunnel) may initiate the condensation of water vapor into tiny droplets. The latent heat released has non-negligible effect on the structure of a considered flow field. This contribution presents the model for humid air flow at high velocities, which is based on the former model [3] originally developed for the simulation of wet steam flow.

Model of humid air flow

Development of numerical algorithms for flow simulations is not possible without relevant experimental data. Some cases, e.g. transonic flow in a turbine cascade, are very sensitive even on minute changes of geometry of flow conditions [2]. The working fluid in experiments is often the air with some remaining humidity. The latent heat released due to condensation may affect the measured data. Numerical simulation of such condensation could help to estimate the measurement uncertainty.

We consider a simplified model of humid air composed of dry air, water vapor and tiny liquid droplets. We consider the same velocity for all components, therefore the flow of the whole mixture can be modeled by set of transport equations

$$\frac{\partial}{\partial t}\big(A(x)W_k\big) + \frac{\partial}{\partial x}\big(A(x)F_k\big) = P_k + A(x)Q_k, \tag{1}$$

where $A(x)$ is the cross-sectional area of the nozzle and the first three components of W_k, F_k, P_k and Q_k are

$$\begin{bmatrix} \rho \\ \rho u \\ e \end{bmatrix}, \quad \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (e+p)u \end{bmatrix}, \quad \begin{bmatrix} 0 \\ pA'(x) \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{2}$$

with ρ, u, p and e denoting the mixture density, velocity, pressure and total energy per unit volume respectively. We consider the non-equilibrium model for the condensation of water vapor in the form of transport equations for the moments of droplet spectra $Q_n = \int_0^\infty r^n N(r)dr$ with distribution function $N(r)$. The remaining components of W_k, F_k, P_k and Q_k are

$$\begin{bmatrix} \rho\chi \\ \rho Q_2 \\ \rho Q_1 \\ \rho Q_0 \end{bmatrix}, \quad \begin{bmatrix} \rho\chi u \\ \rho Q_2 u \\ \rho Q_1 u \\ \rho Q_0 u \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} \frac{4}{3}\pi r_c^3 J\rho_l + 4\pi\rho G_2\rho_l \\ r_c^2 J + 2\rho G_1 \\ r_c J + \rho G_0 \\ J \end{bmatrix}, \tag{3}$$

with χ, J and r_c denoting the mass fraction of liquid phase, nucleation rate and critical radius of droplet. The term $G_n = \int_0^\infty r^n N(r)g(r)dr$ models the growth of existing droplets with droplet growth velocity $g(r)$, for more details see [1, 3]. The considered system of equations (1) has the same form as the model for wet steam flow. The main differences are hidden in closing relations [1]. The composition of humid air is specified by the maximum mass fraction χ_{max} of water in both gaseous and liquid phases.

Results of numerical simulations

The first tests have been performed by the in-house finite difference Lax-Friedrichs method. The latent heat release was implemented by the 'switch' model, i.e. we solve only first three transport equations and the vapor jumps from metastable state into the equilibrium state in the point x_N, where specified vapor sub-cooling is reached

$$\chi = \begin{cases} 0, & x < x_N, \\ \chi_{eq}, & x \geq x_N, \end{cases} \tag{4}$$

Such simple model is able to approximate the pressure of pure steam ($\chi_{max} = 1.0$) along nozzle axis, see the Fig. 1. The experimental data are from [4]. The second graph with Mach number distribution along the nozzle axis show the effect of condensation in humid air flow with $\chi_{max} = 0.18$.

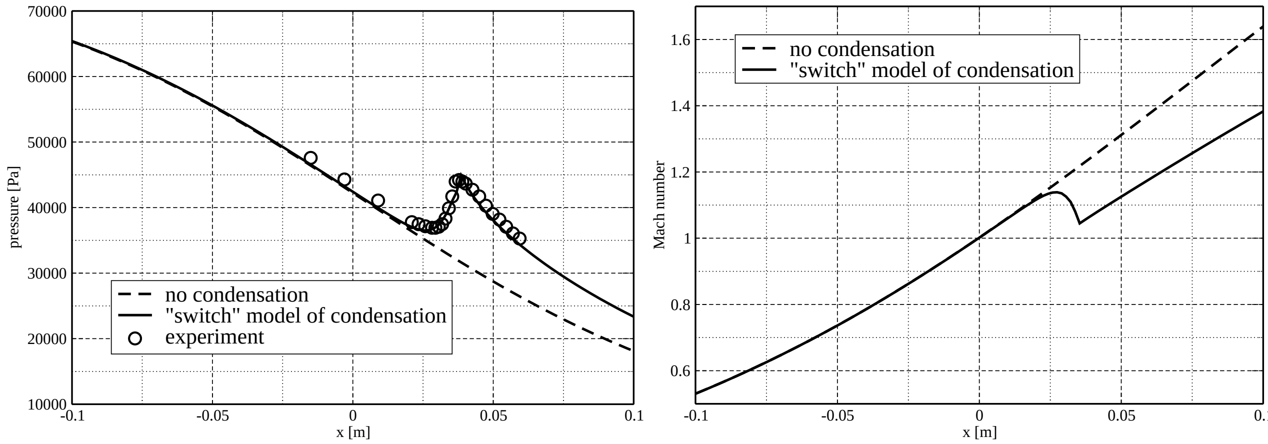

Fig. 1: Pressure in pure steam flow (left) and Mach number in humid air flow (right) along the axis of Barschdorff nozzle [4].

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