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Uncertainty Analysis of Earth Dam

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Abstract: In this contribution we focus on the practical application of the uncertainty propagation in groundwater flow environment [1] using stochastic finite element method [2,3,4], where the uncertain part is taking place in the spatial distribution of the transport properties. From the engineering point of view, it is crucial to choose an optimum between the safety and costs. These criteria are affected by the combination of technology, knowledge and experience of engineers. The essential parts in the prediction of system behavior is the structure geometry and soil properties, represented here by the earth dam with its subsoil and diffusion coefficient, respectively. Such coefficient is usually determined either by the expert guess or very few test drills, whereas the majority of the subsoil properties remains uncertain. The aim of this contribution is to obtain more information and reduce the risks in the system by introducing a stochastic expansion of the underlying deterministic problem.

Introduction

Uncertainty propagation have become popular approach to investigate and predict behaviour of various systems [4,5]. The progress of computer technology then enables to solve more complex models and consequently to get more information about system itself. Such a complex problem is here represented by the extension of deterministic material model with uncertain inputs, which may take origin in the lack or inaccuracy of measurements.

Groundwater flow model

The system is governed by diffusion equation in following form

$$\begin{aligned} -\nabla_{\mathbf{x}} \cdot (\kappa_m(\mathbf{x},\mathbf{y})\nabla_{\mathbf{x}} u(\mathbf{x},\mathbf{y})) &= \widetilde{f}(\mathbf{x}), && \mathbf{x} \in D,\ \mathbf{y} \in \mathbb{R}^m, \\ \mathbf{n}(\mathbf{x}) \cdot (\kappa_m(\mathbf{x},\mathbf{y})\nabla_{\mathbf{x}} u(\mathbf{x},\mathbf{y})) &= \widetilde{f}_N(\mathbf{x}), && \mathbf{x} \in \partial D_N,\ \mathbf{y} \in \mathbb{R}^m, \\ u(\mathbf{x},\mathbf{y}) &= \widetilde{f}_D(\mathbf{x}), && \mathbf{x} \in \partial D_D,\ \mathbf{y} \in \mathbb{R}^m, \end{aligned} \tag{1}$$

where the uncertain term is introduced by material parameter $\kappa_m(\mathbf{x},\mathbf{y})$ [m/s], which in our case consists of log-normally distributed random variables. Spatial discretization is conducted using finite elements and Karhunen-Loève expansion [2] with exponential covariance core is used for stochastic discretization.

The relevant tasks in terms of reliability of such structures are basically the probability of occurrence and position of seepage point and probability of exceeding a critical value of velocity for certain type of soil. With a little additional effort, one can reconstruct the probability density function of the solution in each node of finite element mesh which represents a transformation of random inputs by given equation Eq. 1. Aforementioned relevant quantities can be therefore derived from the probability density function of the solution.

One can also derive basic statistical moments of the solution, i.e. the variance and mean of hydraulic heads, see Fig. 1 and Fig. 2 respectively.

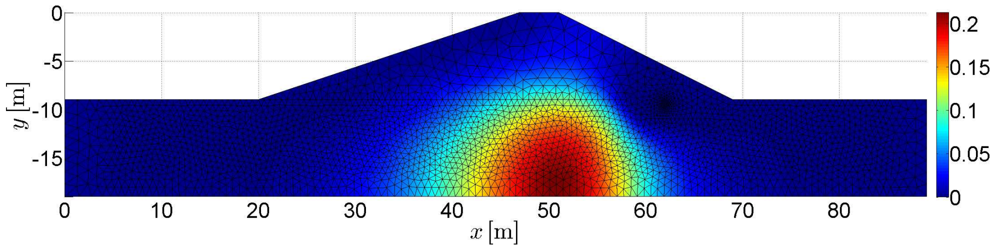

Fig. 1: Variance of hydraulic heads u [m^2]

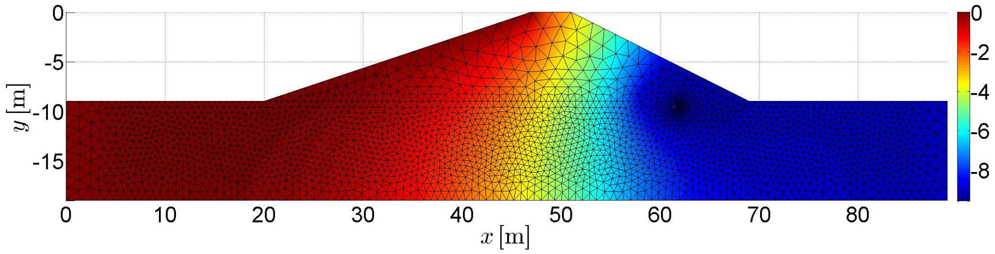

Fig. 2: Mean of hydraulic heads u [m]

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