21st International Conference ENGINEERING MECHANICS 2015
Svratka, Czech Republic, May 11 –14, 2015
Paper #081, pp. 118–119

Parameter Identification of Heterogeneous Materials from a Set of Destructive Experiments

Eliška Janouchová[a], Anna Kučerová[b], Jan Sýkora[c]

Faculty of Civil Engineering, CTU in Prague; Thákurova 7/2077; 166 29, Prague; CZ

[a]eliska.janouchova@fsv.cvut.cz, [b]anicka@cml.fsv.cvut.cz, [c]jan.sykora.1@fsv.cvut.cz

Keywords: uncertainty quantification, heterogeneous materials, uncertain parameter identification, stochastic inversion problem, principal component analysis, Markov chain, Monte Carlo sampling

Abstract: Structural parameters such as material properties involve uncertainties which need to be considered in an appropriate reliability analysis. The contribution focuses on stochastic parameter estimation of heterogeneous materials based on observations from a set of destructive experiments.

Introduction

Heterogeneous character of building materials causes spatial variations of mechanical parameters affecting the structural system behaviour under the loading. This phenomenon can be observed during laboratory testing on a set of specimens made of the same material. In order to investigate the structural reliability properly, these inherent uncertainties in material parameters have to be captured by a corresponding stochastic model. Calibration of such a model can be formulated as a search for probabilistic description of its parameters providing the distribution of the model response corresponding to the distribution of the observed data, i.e. a stochastic inversion problem.

Uncertainties can be divided into two main categories according to whether a source of nondeterminism is irreducible or reducible [1]. Our goal is to quantify aleatory (irreducible, inherent, stochastic) uncertainty which corresponds to real variability of properties in the heterogeneous material, while epistemic (reducible, subjective, cognitive) uncertainty arising from our lack of knowledge is supposed to be reduced by any new measurement according to the coherence of learning [2].

Inverse problems are often ill-posed, because the function mapping the parameters to the responses is not injective so it is also non-invertible. The most common method of parameter estimation is based on minimisation of the difference between the experimental data and the model response. In the last decades probabilistic methods to modelling of uncertainties have became applicable thanks to a growing computational capacity. The probabilistic approach restates the inverse problem as well-posed in an expanded stochastic space by modelling the parameters as well as the observations as random variables with their probability distributions [3]. Several methods for uncertainty quantification in probabilistic settings have been proposed in the literature. They are mainly based on choosing a particular type of probability density function (pdf) of variables and the corresponding parameters of this distribution are provided by e.g. maximum likelihood method [4] or Bayesian inference [5]. Authors in [6] represent random variables by a polynomial chaos expansion and identify its coefficients from the data. An extensive overview on stochastic modelling of uncertainties can be found in [7].

This contribution concentrates on stochastic parameter identification of heterogeneous materials. The proposed method quantifying the aleatory uncertainties concentrates on using a probabilistic method to represent the source of uncertainty and then Markov chain Monte Carlo (MCMC) sampling [8] to compute the moments of the parameters' distribution through a deterministic model.

Stochastic identification method

The developed method is initially inspired by the Bayesian inference where all the available information are combined in resulting updated distribution of the parameters. The underlying problem of its practical application to real experimental data is an appropriate formulation of the a priori pdf of parameters and function representing the distribution of experimental measurements. As it is focused on the quantification of irreducible uncertainty in the data, the influence of the prior information is suppressed and an non-informative uniform a priori pdf is employed.

We consider a situation where available observations are obtained within a set of destructive experiments, each performed on a different set of specimens. Simple multiplication of pdfs constructed for data from each experiment may lead to underestimating the parameter variance due to underlying assumption of independence among observations from different experiments. Despite the independence of specimens, the observations may be correlated due to their physical meaning described by the material model and hence their dependence on the same material parameters.

The proposed methodology is based on transformation of the data by principal component analysis (PCA) into a set of independent quantities [9]. The PCA requires information about mutual correlations among data from particular experiments, which needs to be estimated using the prior knowledge about the underlying model. Considering only a reasonable number of the principal components in stochastic model updating allows eliminating the influence of experimental errors to a certain extent.

Summary

The contribution is focused on developing a method for identification of parameters along with their variations in heterogeneous materials from a set of experiments. The procedure is based on MCMC sampling of a combination of the non-informative prior pdf and PCA-based pdf of experimental data where correlations between particular observed variables and measurement errors are eliminated.

Acknowledgement: This outcome was financially supported by the Czech Science Foundation, project No. 15-07299S, and the Grant Agency of the Czech Technical University in Prague, grant No. SGS15/030/OHK1/1T/11.

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