Nonlinear Behaviour of Concrete Foundation Slab
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**Abstract:** A layered model is used for nonlinear analysis of a foundation concrete slab. Calculation is performed using interaction with elastic Winkler-Pasternak subsoil model and considering plastic yielding of slab layers. Two Drucker-Prager yield criterions define a nonlinear material model for concrete. Computation is done by SIFEL solver using the Finite Element Method.

**Introduction**
The contribution is dedicated to nonlinear analysis of concrete foundation slab. Computation is performed by SIFEL [1], a specialized software package used for solving mechanical and transport problems and being developed at Faculty of Civil Engineering, Czech Technical University in Prague. SIFEL has been already used for the solution of foundation slab problem [2] but it has been extended by two new material models, a concrete plasticity model using two Drucker-Prager (DP) yield conditions and a layered model for plate/shell structures. These two material models are briefly described in this paper.

**Double Drucker-Prager concrete plasticity model**
By the joining of two DP yield conditions [3], a plasticity material model that solves space stress states can be obtained. Using proper parameters one of the conditions is set to describe tensile behaviour of concrete and the other condition is intended to approximate concrete in compression. General DP yield function is defined as follows

\[ f(\sigma) = \alpha_\phi I_1(\sigma) + \sqrt{J_2(\sigma)} - \tau_0 \] (1)

where \( \alpha_\phi \) and \( \tau_0 \) are the parameters of the model. \( \tau_0 \) represents the shear strength and \( \alpha_\phi \) is connected with friction angle. \( I_1 \) is the first invariant of stress tensor and \( J_2 \) is the second invariant of deviatoric stress tensor. This yield condition creates a cone if it is displayed in the principle stress space.

Each yield condition is set by using parameters \( \alpha_\phi \) and \( \tau_0 \) according to which stress area it is supposed to describe [4]. These parameters are related to strength in tension and compression of concrete. The problem of stress return areas and first derivative singularities, which are located in the intersection of DP conditions and at the top of the cone, is solved on the level of the stress invariants. Softening procedure is added to the model for the tensile stress area and, in the case of plane stress problem, a special treatment for the transverse stress component is applied.

**Layered model for plate/shell structures**
To simplify calculation, a general plate/shell structure may be represented by its middle plane. By usage of a layered model [5], material properties across the thickness of a structure are took into account without making of full 3D topology. The layered model divides structures into small layers where each layer is considered to be in the plane stress state and is given its own values of
deformation \((e_x, e_y, \gamma_{xy})\) and stress \((\sigma_x, \sigma_y, \tau_{xy})\). Deformation and stress components of the j-th layer are driven by the following equations

\[
e_j = e_0 + z_j \kappa, \quad \sigma_j = D_j e_j,
\]

where \(e_0\) represents the deformation of the middle plane, \(\kappa\) stands for the bending curvatures, \(z_j\) determines the distance of the j-th layer from the middle plane and \(D_j\) is considered to be a stiffness matrix for the plane stress state. All contributions to stress resultant forces from individual layers are added and constitutive relation between stress resultant forces and deformations can be written

\[
\begin{bmatrix}
    n \\
    m
\end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix}
    t_j D_j & z_j t_j D_j \\
    z^2_j t_j D_j & \kappa
\end{bmatrix} \begin{bmatrix}
    e_j \\
    \kappa
\end{bmatrix},
\]

where \(t_j\) denotes the thickness of the j-th layer, \(n\) and \(m\) are the stress resultant forces.

The model is implemented into SIFEL solver in such way that there is a possibility to assign different material models to each layer and to perform a nonlinear calculation with them. For instance, it is possible to compute plasticity analysis of structure considering various yield conditions for layers.

Application of the models

The above described models can be used in simulation of reinforced concrete slabs, walls or shells where concrete layers may be defined by the double DP model and reinforcement layers may be directed by J2 plasticity. The implementation of the presented material models was motivated by a real engineering problem of a foundation slab in a storage hall exposed to a concentrated load. This problem has been analysed with help of the implemented models and the results from the simulation together with all further findings will be presented at the conference contribution.

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References


