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# Application of the DOProC Method in Solving Reliability Problems

Martin Krejsa[a*], Petr Janas[b], Vlastimil Krejsa[c]

VSB–Technical University of Ostrava, Faculty of Civil Engineering, Department of Structural Mechanics, Ludvika Podeste 1875/17, 708 33 Ostrava–Poruba, Czech Republic
[a]martin.krejsa@vsb.cz, [b]petr.janas@vsb.cz, [c]vlast.krejsa@seznam.cz

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**Abstract:** The Direct Optimized Probabilistic Calculation (DOProC) is originally developed as one way of solving probabilistic problems which don't use any simulation technique. DOProC is based on general terms and procedures used in probabilistic theories. Input random quantities (such as the load, geometry, material properties, or imperfections) are in DOProC method expressed by the parametric or empirical distribution in histograms. DOProC applications are processed in ProbCalc software, in which the calculation model under analysis can be expressed analytically as a sign arithmetic expression or can be expressed using code from the dynamic library. The method requires high-performing information systems for complex tasks. Therefore, efforts have been made to optimize calculations in order to reduce the number of operations, keeping, at the same time, reliable calculation results. The paper will also focus on other special software applications that are able to assess the reliability of bearing structures with respect to fatigue damage or underground works with anchors.

## Introduction

Calculation model can be defined in probabilistic tasks in general as the function of $n$ random variables $X_1, X_2 \ldots X_n$, which can be mutually statistically dependent and independent. Resulting variable $Z$, expressed generally as:

$$Z = f(X_1, X_2 \ldots X_n) \ , \tag{1}$$

is also random variable, which may be expressed as statistical moments, parametric probability distribution or empirical probability distribution in form of nonparametrically defined histogram.

The structure must be designed so that the resistance of the structure $R$ is greater than the load effect $S$. Taking into account all randomness in loads, manufacturing and assembly imperfections and the environment properties in which designed structure performs its function, resistance $R$ and load effect $S$ is to be considered as random variables. In the case of probabilistic assessment of the structure is the function of the random variables under analysis Eq. 1 safety margin, defined e.g. as:

$$Z = R - S \ . \tag{2}$$

Common notation of the estimated failure probability $p_f$, relative to the criterion of reliability, is defined as:

$$p_f = P(R < S) = \int_{D_f} f(X_1,\, X_2 \ldots X_n)\, \mathrm{d}X_1,\, \mathrm{d}X_2 \ldots \mathrm{d}X_n \ , \tag{3}$$

where $D_f$ is failure area of the safety margin $Z(\mathbf{X}) < 0$, a $f(X_1,\, X_2 \ldots X_n)$ as the function of joint probability density of random variables $\mathbf{X} = X_1,\, X_2 \ldots X_n$.

Determination of failure probability $p_f$ based on the explicit calculation of the integral Eq. 3 is generally unmanageable. For solutions have been developed and continues to develop a series of stochastic methods (simulation, approximate - see [1]). The proposed method: Direct Optimized Probabilistic

Calculation - DOProC, which is developed since 2002, solves the integral Eq. 3 pure numerical way that is based on basis of probability theory and does not require any simulation technique. This is highly effective way of probabilistic calculation in terms of computation time and accuracy of the solution. The novelty of the proposed method lies in an optimized numerical integration. In summary was published in [2].

## DOProC Method - theory

Similar to many other probabilistic methods, the non-parametric (empirical) distribution of input random quantities in DOProC, such as the load, geometry, material properties, or imperfections, are expressed by means of histograms. It is also possible to use parametric distributions, typically based on observations, often of long-term data [3].

In probabilistic tasks are input random variables often statistically dependent - for example xross-section properties, strength and stiffness characteristics of the materials. In the calculations carried out by DOProC method can be statistically dependent input random variable expressed by the so-called multidimensional histograms (double, triple) [4].

Mathematical description of the basic computational algorithm of the DOProC method is based on a calculation of the probability $b^{(i)}$ of resulting random variable $B$, which is discretized to $i = 1 \ldots n$ intervals:

$$p_b^{(i)} = \sum_{l=1}^{l} p_{bl}^{(i)} = \sum_{l=1}^{l} \left( p_{aj}^{(i1)} \cdot p_{aj}^{(i2)} \cdot p_{aj}^{(i3)} \cdot \ \ldots \ \cdot p_{aj}^{(ij)} \cdot \ \ldots \ \cdot p_{aj}^{(in)} \right) , \tag{4}$$

where value $b^{(i)}$ is resultant probability $\left(a_1^{i1}, a_2^{i2} \ldots a_j^{ij} \ldots a_n^{in}\right)_l$ of function $f$ with input random variables $A_j$ of $i_j$ intervals. The procedure above was comprehensively published, e.g. in [5].

Number of intervals $i_j$ of each histogram $A_j$ can be different as well as the number of intervals $i$ of histogram $B$. The number of intervals $N$ with the total number of input random variables $n$ is for the number of necessary arithmetic operations and necessary calculation time the most decisive factor. It also significantly affects the accuracy of probabilistic calculation. If there are too many random quantities, the tasks require too much time even if advanced computational facilities are available. Therefore, efforts have been made to optimize calculations in order to reduce the number of operations, while retaining reliable calculation results:

**The grouping of variables.** This procedure can be used e.g. in situations where the random variable input or output variables can be expressed using one joint histogram. This leads to a large reduction of computational operations.

**Parallelization.** The calculation algorithm of DOProC method is advantageous for use on machines with two or more CPUs or their cores. The basic computational algorithm of DOProC Eq. 4 can be divided the number of computational operations up to as many parts as there are available execution units, and after partial calculations can be put together from partial results into the histogram of resulting variable, e.g. histogram of safety margin $Z$.

**Interval optimizing.** The purpose of this computational procedure is to reduce the intervals of each variables involved in the calculation. Input random variables don't affect the outcome of the probabilistic calculation as well - are differently sensitive. For input variables that affect the outcome probability less, therefore the number of classes can be reduced. Custom probabilistic calculation is then carried out with the minimum number of intervals for each input random variables.

**Zone optimizing.** The intervals of each individual histogram are clearly defined during the calculation using one to three types of zones, depending on influence on resulting probability of failure (contribute always, may or may not contribute, contribute never). The calculation then will be limited only on intervals of input random variables which clearly don't contribute the resulting value of failure probability.

**Trend optimizing.** This optimization of probabilistic calculation follow the zonal optimizing. This optimization of probabilistic calculation determines the trends of changes in the histograms of input variables when defining individual zones.

Such procedures can be combined, thereby achieving an even stronger acceleration of the calculation. Computational procedures have been described were comprehensively published in [6, 7, 8].

## DOProC Method - applications

The algorithm of DOProC method has been implemented in several software applications, and has been used several times in probabilistic tasks and probabilistic reliability assessments. For the application of the DOProC method can be used programming system ProbCalc, in which it is relatively easy to implement analytical and numerical transformation probabilistic model of solved tasks [9, 10]. The program system ProbCalc is extensively usefull in solving of probabilistic tasks of engineering practice, especially on probabilistic reliability assessment according to the current standards [11].

**Probabilistic calculation of fatigue cracks in steel structures under cyclical loads.** In [12, 13, 14] was published in detail the methodology for probabilistic assessment of structures exposed to fatigue, focusing on the determination of acceptable size of fatigue crack and definition of the regular inspection system. This relatively advanced probabilistic task was solved using ProbCalc, but also using new application under development titled FCProbCalc [15], which allows in a user friendly environment to calculate the probability of fatigue crack progression.

**Probabilistic reliability analysis of anchor reinforcement.** The comprehensively methodology for probabilistic design and reliability assessment of anchor reinforcement in long mining and underground works was utilized. It was also established a program Anchor, with which is possible to realize the probabilistic calculation very flexibly [16].

## Summary

The paper introduces the development of probabilistic methods with particular attention to a new method, DOProC, which is still under development. DOProC appears to be a very efficient method and the solution suffers only from numerical errors and those errors resulting from discretizing of input and output quantities. One shortcoming of DOProC is the considerable increase in the required computer time for probabilistic operations with many random variables in the model. The maximum number of random variables depends on the complexity of this model, and importantly whether it is possible to use any of the described optimization steps. Also the developed program tools implementing the DOProC method were stated and which are at present able to solve many challenges in probabilistic calculations.

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## References

[1] R. E. Melchers, Structural Reliability Analysis and Prediction, Second edition, John Wiley & Sons Ltd., England (1999) 437 p, ISBN: 0-471-98324-1.

[2] P. Janas, M. Krejsa, V. Krejsa, J. Sejnoha, Direct Optimized Probabilistic Calculation — DOProC, submitted to Probabilistic Engineering Mechanics (2014).

[3] R. Cajka, M. Krejsa, Measured data processing in civil structure using the DOProC method, Advanced Materials Research, 859 (2014) 114–121, DOI: 10.4028/www.scientific.net/AMR.859.114.

[4] P. Janas, M. Krejsa, Statistical Dependence of Input Variables in DOProc Method, Transactions of the VSB – Technical University of Ostrava, Civil Engineering Series, 12 (2012) 48–58, ISSN: 1804-4824, DOI: 10.2478/v10160-012-0017-3.

[5] P. Janas, M. Krejsa, I. Kolos, Reliability assessment of structure expressed to impact, in: Proceedings of conference Engineering Mechanics 2003, The Institute of Theoretical and Applied Mechanics AS CR (2003) ISBN: 80-86246-18-3. (In Czech)

[6] P. Janas, M. Krejsa, V. Krejsa, Contemporary Capability of Direct Determined Probabilistic Calculation, in: Proceedings of conference Engineering Mechanics 2006, The Institute of Theoretical and Applied Mechanics AS CR (2006) p 132–33, ISBN: 80-86246-27-2.

[7] P. Janas, M. Krejsa, V. Krejsa, Structural reliability assessment using a Direct Determined Probabilistic Calculation, in: Proceedings of the 12th Conference on Civil, Structural and Environmental Engineering Computing, Civil-Comp Press (2009) DOI: 10.4203/ccp.91.72.

[8] P. Janas, M. Krejsa, V. Krejsa, Using the Direct Determined Fully Probabilistic Method for determination of failure, in: European Safety and Reliability Conference ESREL 2009, Reliability, Risk and Safety: Theory and Applications. Taylor & Francis Group, London (2010) p 1467–1474, ISBN: 978-0-415-55509-8, DOI: 10.1201/9780203859759.ch203.

[9] M. Krejsa, P. Janas, V. Krejsa, Software Application of the DOProC Method, International Journal of Mathematics and Computers in Simulation, 8 (2014) 121–126, ISSN: 1998-0159.

[10] M. Krejsa, P. Janas, V. Krejsa, ProbCalc - An efficient tool for probabilistic calculations, Advanced Materials Research, 969 (2014) 302–307, DOI: 10.4028/www.scientific.net/AMR.969.302.

[11] M. Krejsa, P. Janas, R. Cajka, Using DOProC Method in Structural Reliability Assessment, Applied Mechanics and Materials, 300–301 (2013) 860–869, DOI:10.4028/www.scientific.net/AMM.300-301.860.

[12] Krejsa, M., Stochastic Modelling of Fatigue Crack Progression using the DOProC Method, in: Proceedings of the 11th International Conference on Computational Structures Technology, Civil-Comp Press (2012) DOI: 10.4203/ccp.99.113.

[13] M. Krejsa, Probabilistic failure analysis of steel structures exposed to fatigue, Key Engineering Materials, 577–578 (2013) 101–104, DOI: 10.4028/www.scientific.net/KEM.577-578.101.

[14] M. Krejsa, Probabilistic reliability assessment of steel structures exposed to fatigue, Safety, Reliability and Risk Analysis: Beyond the Horizon, CRC Press-Taylor & Francis Group (2014) 2671–2679, ISBN: 978-1-315-81559-6; 978-1-138-00123-7, DOI: 10.1201/b15938-404.

[15] M. Krejsa, The Probabilistic Calculating of Fatigue Crack Propagation Using FCProbCalc Program, in: Proceedings of 18th International Conference Engineering Mechanics 2012, The Institute of Theoretical and Applied Mechanics AS CR (2012) p 745–754, ISBN: 978-80-86246-39-0.

[16] M. Krejsa, P. Janas, I. Yilmaz, M. Marschalko, T. Bouchal, The Use of the Direct Optimized Probabilistic Calculation Method in Design of Bolt Reinforcement for Underground and Mining Workings, Scientific World Journal, Vol. 2013 (2013) DOI: 10.1155/2013/267593.