

Solution of Large Engineering Problems on Parallel Computers

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Abstract: This contribution reviews utilization of parallel computers in engineering problems. Solution of systems of algebraic equations is usually the most demanding part of an analysis which could be accelerated by domain decomposition methods. Other application of parallel computers in engineering simulations contains multi-scale problems where different processors solve different levels of the problems.

Introduction

Although hardware is permanently being developed, requirements on computer power and memory arising from engineering practice exceed usually its possibility. Single processor computers are definitely unable to solve large problems. Therefore parallel computers are becoming very popular and they are used not only in academic institutions but also in engineering practice.

Parallel computers could be sorted out with respect to their memory. The computers with shared memory are called massively parallel computers. All processors have access to all data. On the contrary, a cluster of single-processor computers connected via suitable network is a parallel computer with distributed memory. Each processor has an access only to data stored in its memory. If a processor A needs data stored in memory of processor B, the data has to be sent. The massive parallel computers are significantly more expensive in comparison with clusters. The biggest parallel computers at this time are clusters of multi-processor nodes and the memory is distributed while in nodes is shared.

Classification of problems

There are four basic groups of demanding computations. First group contains problems with many unknowns (more than million) which are solved only once. Such problems can be split into smaller subproblems which are distributed to processors of a parallel computer and they are solved independently there. At the end, the original continuity on subproblem interfaces is enforced. This is the principle of domain decomposition methods [1, 2, 3]. Their parallelization requires data transfer among processors, synchronization of operations and load balancing.

Second group contains problems with moderate number of unknowns (tens of thousands) which are solved repeatedly and the order of solution is not important or it is important only partially. The Monte Carlo simulations, various optimization tasks or multi-scale analysis [4], where a problem defined on the lower level is solved for every finite element of the upper level can serve as examples. Parallelization of such computations can lead to the ideal speedup because the master processor sends and receives only small amount of data. The same number of operations executed on the slave processors leads to the ideal load balancing which is another advantage.

Third group contains problems with a moderate number of unknowns which are solved repeatedly and the order of solution is important. As an example can serve a numerical time integration. Parallelization of such problems is questionable. The only solution is simultaneous space-temporal

discretization where the number of unknowns grows significantly in comparison with the classical approach but the resulting system of equations is solved only once. Methods of the first group mentioned in this contribution can be used.

Fourth group contains problems with many unknowns which should be solved repeatedly. These are not solvable at this time.

Examples

Two domain decomposition methods will be described in detail, namely the Schur complement method [5] and the FETI (Finite Element Tearing and Interconnecting) method [6, 7]. Behaviour of the methods will be documented on several examples solved. Second group of problems will be represented by a two-level analysis of coupled heat and moisture transport in a masonry bridge structure [4]. The coupled transport is described by the Künzel model. The bridge is described by two and three dimensional models and two different meso-scale levels are used because of heterogeneous masonry.

References

- [1] A. Toselli, O. Widlund, *Domain Decomposition Methods--Algorithms and Theory*, Springer-Verlag, Berlin, 2005.
- [2] A. Quarteroni, A. Valli, *Domain Decomposition Methods for Partial Differential Equations*, Clarendon Press, Oxford, 2005.
- [3] J. Kruis, *Domain Decomposition Methods for Distributed Computing*, Saxe-Coburg Publications, Kippen, Stirling, 2006.
- [4] J. Sýkora, T. Krejčí, J. Kruis, M. Šejnoha, Computational homogenization of non-stationary transport processes in masonry structures, *Journal of Computational and Applied Mathematics*, 236 (2012) 4745–4755.
- [5] J. Kruis, Domain Decomposition Methods on Parallel Computers, in: B.H.V. Topping, C.A. Mota Soares (Eds.), *Progress in Engineering Computational Technology*, Saxe-Coburg Publications, 2004, 299–322.
- [6] Z. Dostál, *Optimal Quadratic Programming Algorithms With Applications to Variational Inequalities*, Springer Verlag, 2009.
- [7] J. Kruis, The FETI Method and its Applications: A Review, in: B.H.V. Topping, P. Iványi (Eds.), *Parallel, Distributed and Grid Computing for Engineering*, Saxe-Coburg Publications, 2009, 199–216.