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Analysis of the Boundary Problem with the Preference of Mass Flow

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Abstract: We work with the numerical solution of the turbulent compressible gas flow, and we focus on the numerical solution of these equations, and on the boundary conditions. In this work we focus on the outlet boundary condition with the preference of given mass flow. Usually, the boundary problem is being linearized, or roughly approximated. The inaccuracies implied by these simplifications may be small, but they have a huge impact on the solution in the whole studied area, especially for the non-stationary flow. The boundary condition with the preference of mass flow is sometimes being implemented with the use of some iterative process, guessing the correct values (for the pressure, density, velocity) in order to match the given mass flow through the boundary. In our approach we try to be as exact as possible, using our own original procedures. We follow the exact solution of the initial-value problem for the system of hyperbolic partial differential equations. This complicated problem is modified at the close vicinity of boundary, where the conservation laws are supplied with the additional boundary conditions. We complement the boundary problem suitably, and we show the analysis of the resulting uniquely-solvable modified Riemann problem. The resulting algorithm was coded and used within our own developed code for the solution of the compressible gas flow (the Euler, NS, and RANS equations). The examples show good behaviour of the analyzed boundary condition.

The problem

The main problem of this boundary condition is to solve the following system (conservation laws in 1D), equipped with one-side initial condition and the complementary condition (preference of mass flow at the boundary).

$$\begin{aligned} &\frac{\partial \varrho}{\partial t} + \frac{\partial u\varrho}{\partial x} = 0 && \varrho(x,t) = \varrho_L, \quad x<0 \\ &\frac{\partial \varrho u}{\partial t} + \frac{\partial (p+\varrho u^2)}{\partial x} = 0 && u(x,t) = u_L, \quad x<0 \qquad \varrho(0,t)u(0,t) = G_\star \\ &\frac{\partial \varrho(\varepsilon + \frac{1}{2}u^2)}{\partial t} + \frac{\partial(\varrho u(\varepsilon+\frac{1}{2}u^2)+pu)}{\partial x} = 0 && p(x,t) = p_L, \quad x<0 \end{aligned} \tag{1}$$

Here ϱ is the density, p pressure, u velocity, $\varepsilon = p/\varrho(\kappa-1)$ is the specific internal energy, κ is the adiabatic constant, $G_\star$ is prescribed mass flow (per unit area). This problem has a unique (entropy weak) solution for the reasonable input data $\varrho_L, u_L, p_L, \kappa, G_\star$. The complete analysis of this problem is based on the solution of the Riemann Problem for the Euler equations.

Example

The developed software with presented boundary condition was used for the simulation of the compressible turbulent flow in the 3D axis-symmetrical channel. Axis x is the axis of symmetry. At the geometry crosscut shown at Fig. 1., the inlet is located at the upper part of the boundary, the outlet is located right. The computational mesh in 2D crosscut consisted of 73x97 quadrilaterals. The initial condition was formed by the state $\theta^o = 273.15,\ v_1^o = 0,\ v_2^o = 0,\ v_3^o = 0,\ p^o = 101325$. The boundary condition conserving the total pressure, total temperature, and zero tangential velocity, with $\theta_o = 273.15, p_o = 101325$ was used at the inlet. At the outlet, the boundary condition with preference of mass flow was used, with $G_\star = 4.0$ in average. At each face, the value $G_\star$ was computed (in each iteration) in order to match the average (across the whole boundary). Other boundaries were considered as wall: The boundary condition preffering the zero normal velocity was used in the case of the inviscid flow. For the viscous flow, this condition was modified by the zero velocity at the wall, and wall temperature $\theta_{WALL} = 273.15$ was set. Further $k_{WALL} = 0$ and $\omega_{WALL} = c_\omega \frac{6\mu}{\beta \varrho y_s^2}$, $c_\omega \frac{6}{\beta} = 120$. Here by y_s we mean the distance between the face and the center of the neighbouring element.

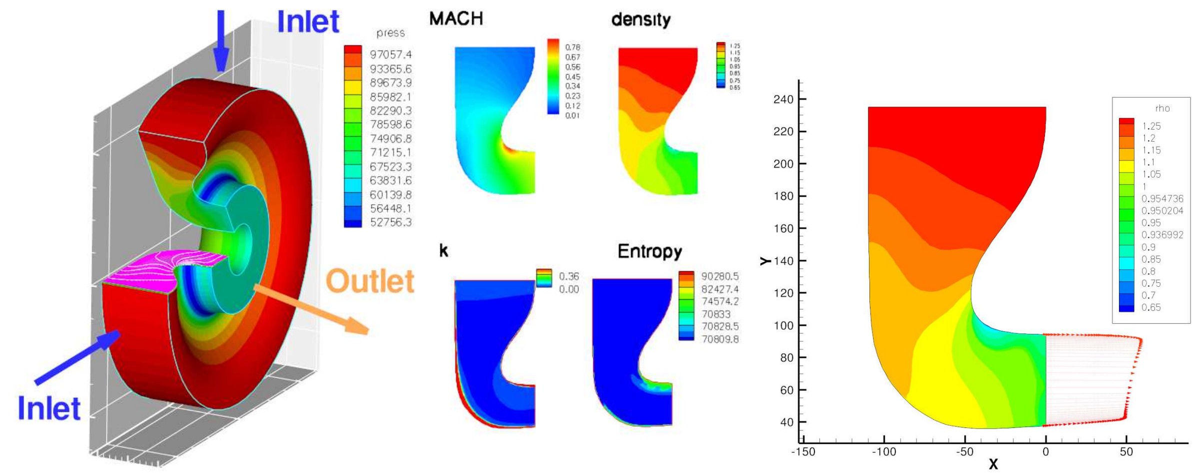

Fig. 1: 3D axis-symmetrical turbulent flow, 3D geometry shape, results at chosen 2D crosscuts.

Summary

This paper shows the analysis of the outlet boundary condition by preference of mass flow, based on the modification of the Riemann problem for the Euler equation.

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