21st International Conference ENGINEERING MECHANICS 2015
Svratka, Czech Republic, May 11 –14, 2015
Paper #157, pp. 206–207

Adaptive Updating of Meta-Models in Hyperspherical Domains

Eva Myšáková[a], Matěj Lepš[b]

Faculty of Civil Engineering, CTU in Prague, Thákurova 7, 166 29, Prague, CZ

[a]eva.mysakova@fsv.cvut.cz, [b]leps@cml.fsv.cvut.cz

Keywords: design of experiments, meta-modeling, design domain, hypercube, hypersphere

Abstract: The meta-modeling is frequently used method for analysis of the complex systems. Its basic part constitutes the design of experiments (DoE) - it is crucial to select the training points with consideration. Therefore the appropriate design domain where the points are to be spread has to be selected. The meta-models are usually constructed within the domain in shape of hypercube. In this contribution we propose the alternative - the hyperspherical design domain.

Introduction

A Design of Experiments (DoE) is a set of the design points whose coordinates correspond to combinations of values of system's input variables [1]. It plays an essential role in meta-modeling which is often used method for analysis of the complex systems' behavior [2]. The purpose of meta-models is approximation of the system response in cases where the evaluation of the original model is costly or highly time-consuming. The meta-model is constructed based on (i) training data - the design points - spread over the domain of real input parameters and (ii) responses of the original model corresponding to those design points. For good meta-model predictions the positions of design points have to be selected carefully. Furthermore the meta-model usually needs to be updated by addition of more training points into appropriate areas of the design domain.

The training points are traditionally generated within the domain in a shape of hypercube. This contribution deals with alternative procedure: the points are placed within the hyperspherical domain - which seems to be effective e.g. for solving reliability-based problems [3]. The additive points are then found using two-objective strategy - the new points are added (i) into the areas that are not still covered well and (ii) into the areas interesting for the response of the meta-model.

Meta-model construction in hypercube

The adaptive updating briefly described in Introduction and discussed for example in [4] originally operates in hypercubical domain. More precisely the domain for the meta-model construction has the shape of a unit hypercube. The domain in a real parameter space is generally a hyperprism because individual parameters differs in their values and distributions. We set the bounds for meta-model construction as 0.15% and 99.85% quantiles ($\mu \pm 3\sigma$ in case of normal distribution; μ is mean and σ is the standard deviation). Outside these bounds the meta-model has to extrapolate the response value. Then uniform space-filling DoE is created in the hypercube, the responses corresponding to the points in real parameter domain are computed by the original model, and the meta-model is constructed. Usually the quality of the meta-model prediction is not precise enough and therefore the meta-model has to be updated by addition of new design points into the DoE.

Motivation

As dimension grows we face problem that the majority of added points are placed on the border of the domain, see Figure 1. It can be caused by the character of the model response - the interesting areas lie outside the selected bounds and therefore the new points are placed as far as possible. The second explanation relates to the shape of the domain. The hypercube simply has too many borders and from

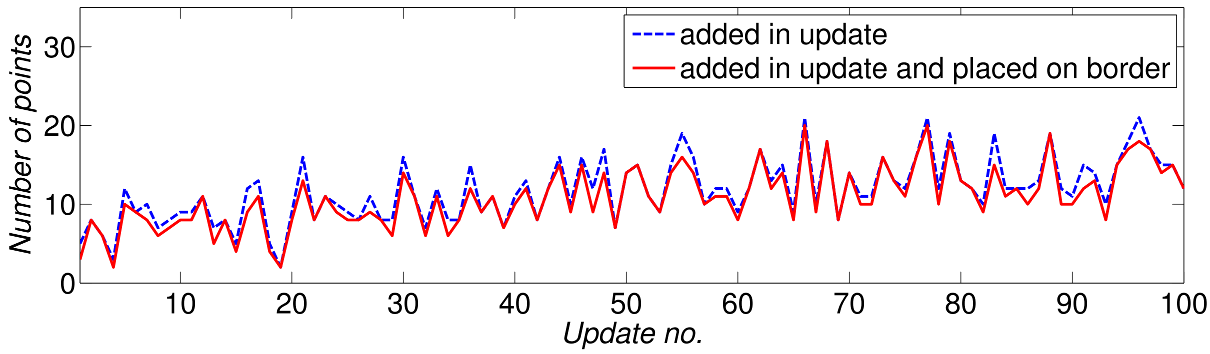

Fig. 1: Points added in individual updates of meta-model constructed within hypercubical domain.

the previous experience it is known that even a good space-filling design of experiments often has the biggest hole on the border object of the design domain (vertex, edge and so forth). Therefore the alternative is proposed - to build the meta-model within the hyperspherical domain.

Hyperspherical design domain

The main idea is to construct the meta-model within the domain in a shape of a hypersphere, which is inscribed in a unit hypercube, because the meta-model usually operates in $[0, 1]^n$ space; n is dimension. In real parameter space the corresponding domain has hyperellipsoidal shape. For simple comparison of the results we set the ellipsoidal semiaxes such that the volume of the hyperellipsoid is same as the volume of the hyperprism in the previous case of hypercubical design domain. So the volume of the real parameter space area covered by the meta-model remains the same in both cases. At first the initial design of experiments in the hypersphere is created [5]. This design is generated in a unit hypersphere (unit radius, centered at the origin), therefore it has to be transformed (i) into the real parameter space (hyperellipsoid) to get the responses of the original model and (ii) into the hypersphere (inscribed in a unit hypercube) for meta-model construction. The procedure is tested on examples with different dimension and various parameter distributions.

Acknowledgement: The authors gratefully acknowledge the financial support from the Czech Science Foundation, project No. 15-07299S, and the Grant Agency of the Czech Technical University in Prague, the grant SGS15/030/OHK1/1T/11.

References

[1] D. C. Montgomery, Design and Analysis of Experiments, Wiley, 8 edition, (2012).

[2] A. Forrester, A. Sóbester, A. Keane, Engineering Design via Surrogate Modelling: A Practical Guide, John Wiley & Sons Ltd, (2008).

[3] V. Dubourg, Adaptive surrogate models for reliability analysis and reliability-based design optimization, Ph.D. thesis, Université Blaise Pascal, Clermont-Ferrand, France, (2011).

[4] E. Myšáková, A. Pospíšilová, M. Lepš, Optimized Design of Computer Experiments: A Review, In: Computational Methods for Engineering Technology, (2014).

[5] E. Myšáková, M. Lepš, Uniform Space-filling Design of Experiments in Hypersphere, In: Proceedings of the 5th Conference Nano and Macro Mechanics NMM 2014, (2014).