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Non-Linear Normal Modes in Dynamics – Discrete Systems

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Abstract: The aim of the paper is to inform about main features of Non-linear Normal Modes (NNM) as a powerful tool for investigation of multi-degree of freedom (MDOF) dynamic systems. In particular, it is shown how this concept can be used to investigate forced resonances of non-linear symmetric systems including non-linear localization of vibrational energy. NNMs can provide a valuable tool for understanding essentially non-linear dynamic phenomena having no counterparts in linear theory and which do not enable analysis using linearized procedures. Discrete MDOF systems are considered in this study. A couple of possible approaches are outlined together with some demonstrations of numerical results.

Introduction

The most popular concept of dynamic systems investigation consists in decomposition into linear normal modes (LNM) which serve as independent separable orthogonal space. These modes are energy invariant and therefore motion consisting of a single mode at any instant will correspond only to this mode for all time. The widespread view being very reliable in the linear domain motivated an idea of expansion into the non-linear domain.

The first self-contained theoretical background of extension into the non-linear domain is ascribed to R.M. Rosenberg [1] and later to R. Rand [2]. Massive development of NNM including theoretical background, numerical implementation, applications and propagation in the community of dynamics has been started by Vakakis e.g. [3] or monograph [4], etc. and simultaneously by A.H. Nayfeh, e.g. [5]. The concept of NNM seems to be more and more popular as a powerful tool for investigation of complicated non-linear dynamic systems, see many papers which appeared recently e.g. [6]. Besides analytical studies dealing with qualitative investigation, many computer implementation have been developed and widely used in daily practice of engineering system design. Well known packages are available, e.g. AUTO, MATCONT, CL-MATCONT, etc.

Definition outline for systems with concentrated masses

The NNM of an undamped discrete or continuous system can be regarded as a synchronous periodic oscillation where all material points of the system reach their extreme values or pass through zero simultaneously. When a discrete system vibrates in an NNM the corresponding oscillation is represented by a line in its configuration space which is termed modal line. Linear systems possess straight modal lines since their coordinates are related linearly during a normal mode oscillation. In non-linear systems the modal lines can be either straight or curved. The later cases are typical in non-linear discrete systems since straight non-linear modal lines reflect rare special symmetries of the system.

For discrete systems Rosenberg defined similar NNM that correspond to straight modal lines in configuration space and non-similar NNM that correspond to curved modal curves. While LNM are all similar, non-linear discrete (or discretized continuous) systems possess basically non-similar modes and similar are rather exceptional. Let us demonstrate the concept of similar and non-similar NNM at the autonomous TDOF system:

$$\begin{aligned} \ddot{x}_1 + x_1 + x_1^3 + K(x_1 - x_2)^3 &= 0, \\ \ddot{x}_2 + x_2 + x_2^3 + K(x_2 - x_1)^3 &= 0. \end{aligned} \tag{1}$$

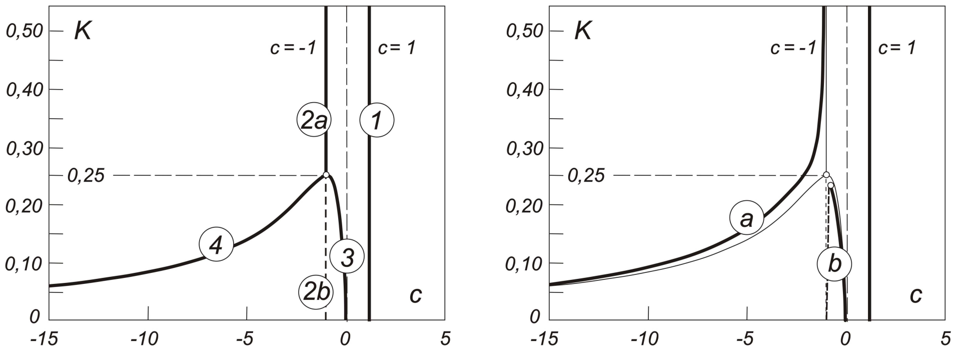

Fig. 1: Proportionality coefficient c as a function of the stiffness K

where K is a symbolic non-linearity rate of the connecting spring. Linear part of this spring is omitted. If the system (1) includes similar NNM a constant parameter c should exist:

$$x_2(t) \;=\; c \cdot x_1(t) \tag{2}$$

Constant c indicates a proportionality of both coordinates x_1, x_2 during every period of eigen vibration and approves an existence of a similar NNM. Fig. 1 - left picture - corresponds with Eqs (1) and shows two similar NNMs - branches 1 and $2a$ (branch $2b$ is unstable) and two non-similar NNMs - branches 3 and 4. Fig. 1 - right picture - presents the case of modified system Eqs (1) which admits slightly different parametr K in both equations and other "non-symmetries". The relevant plots in the right picture demonstrate that purely similar NNMs don't exist any more. The NNMs should be investigated in other way distinguishing global NNMs (close to similar NNMs) and local NNMs (close to non-similar NNMs). In general we can see, that the nonlinear system can include more NNMs than is the number of its degree of freedom.

Conclusion

Let us stress that similar NNM are not generic. Their appearance require quite severe symmetry conditions to be fulfilled and so the similar NNM are rather exceptional in an exact meaning of the term. On the other hand NNM quite often remember shape of a linear counterpart, but anyway their shape changes with level of energy, nodes are moving, etc. These facts makes investigation of NNM more difficult to evaluate, but this concept is still one of the most effective procedure of MDOF nonlinear systems investigation.

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