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# Reconstruction of Random Media via Multi-Objective Optimization

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**Abstract:** To simplify numerical analyzes of complicated real microstructures, a material representation in terms of Representative Volume Element (RVE) has been introduced. It is based on a binary (black and white) image, which statistically resembles the corresponding real microstructure. To characterize microstructure and assess the proximity of the compressed representation to the reference microstructure, several statistical descriptors are usually employed. In one of the approaches to construction of RVE, a unit cell is derived from the optimization procedure formulated in terms of selected statistical descriptors. The aim of our work is to resolve this issue by multi-objective optimization techniques. The goal is to approximate as closely as possible the true Pareto front as a trade-off of competing objectives.

## Introduction

Most of the current multi-scale methods (e.g. $FE^2$) require a substitution of the complicated microstructure (simple examples in Fig. 1) with a relatively small representation, called a statistically equivalent periodic unit cell (SEPUC) [1] that has the same or very similar properties to the original microstructure. A similarity between the original microstructure and its surrogate is appraised e.g. by statistical descriptors. One of the possible procedures how to construct a SEPUC is to utilize an optimization methodology with a statistical descriptor as a part of the optimized objective function. The choice of only one statistical descriptor to capture majority of the microstructure features is not a trivial task and is usually governed by the related computational cost. Formulation of a multi-objective optimization task in which several statistical descriptors are optimized at the same time allows to avoid this difficult selection. Moreover, involving more descriptors strengthen credibility of the representation.

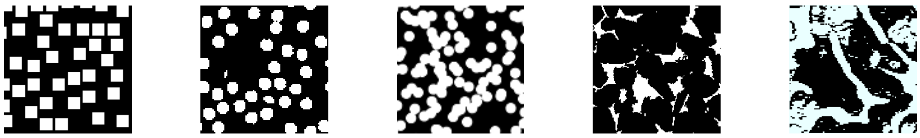

Fig. 1: Examples of optimized microstructures.

Yeong and Torquato [2], for example, optimize chosen statistical descriptors by a simulated annealing, which is single objective optimization algorithm. They use two-point probability and lineal path functions to capture long- and short-range features of the media. The objective function is taken as a weighted sum of the mean square errors between the target descriptors and those corresponding to the reconstructed medium. Here, difficulty lies in the proper adjustments of weights, which influences both the morphology of reconstructed media and complexity of the optimization problem.

## Methodology

In this contribution, the goal is to approximate as closely as possible true Pareto front as a trade-off between competing objectives by a real multi-objective optimization algorithm. The Non-Dominated

Sorting Genetic Algorithm II (NSGA-II) [3] seems to be a suitable methodology for this problem. The NSGA-II is a multi-objective ($\mu + \lambda$) evolutionary algorithm, where the offsprings are selected firstly in accordance with their position on the approximated Pareto front and secondly, with their interpoint (crowding) distance. A new population is created by a special mutation operator based on swapping of two pixels from different phases in a grid. This operator ensures that the volume fraction of all phases remains the same during the whole procedure. In paper [4] is shown, that the swapping based mutation operator is sufficient for this type of optimization algorithms. Therefore, no crossover operator has been used.

For statistical description of the media, we incorporate the same two descriptors as in [2]. Namely, we use the two-point probability function (S2) [5] giving the probability of the ends of a vector with the given length and orientation falling in the same phase. This characteristics is suitable for capturing long-range order correlations. It can be efficiently enumerated with discrete Fast Fourier Transformation making it thus suitable for optimization algorithms. The second objective function, the lineal path function (L2) [5] captures a short-range order characteristic as it states the probability of finding the whole vector being embedded in the same phase. It thus statistically describes the convex part of phase connectivity (in the case of a matrix phase) and the averaged shape of an inclusion (in the case of inclusion phase).

## Conclusions

This contribution serves for a verification of the multi-objective reconstruction of heterogeneous materials described in a pixel form. Since the objective function leading the reconstruction process is multi-modal, a genetic algorithm has been used. The possible multi-modality of the selected statistical descriptors is bypassed by the multi-objective approach, i.e. taking into account more than one statistical descriptor at once.

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