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Forced Vibration Analysis of Timoshenko Beam with Discontinuities by Means of Distributions

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Abstract: The general equations of motion for forced vibration of Timoshenko beam have been used since they were derived assuming there were not any discontinuities in the shear force, the bending moment, the cross-section rotation and the deflection of the beam. However in practice, computing harmonic response of the beam, we encounter concentrated loading, concentrated supports, hinges connecting beam segments, concentrated masses or concentrated moments of inertia all of which may be situated between ends of the beam. The definition of the distributional derivative is chosen in order that all the cases causing jump discontinuities in the shear force or the bending moment or the cross-section rotation can be incorporated. As a result of this approach, more general equations of motion for forced vibration of Timoshenko beam implying all jump discontinuities mentioned are presented in this paper. An analytic closed-form solution to the system of equations is found with integration constants in the form of initial parameters. Making use of this approach, we can find exact expression for the harmonic steady-state response of the uniform beam without summing infinite series and without doing a modal analysis of the beam.

Introduction

Classical analytical method of calculating the harmonic steady-state response of the uniform beam is based on the following main steps [1, 2]. Firstly, we obtain a frequency equation for specific support conditions of the beam. Secondly, we solve the frequency equation for natural frequencies. Thirdly, we find orthogonal mode shapes corresponding to the natural frequencies of the beam. Finally, applying modal analysis, we express the response as a linear combination of the mode shapes by finding corresponding modal participation coefficients.

In this paper, a new analytical method is presented. Applying distributions, it is not necessary for natural frequencies, mode shapes and modal participation coefficients to be computed in analyzing the harmonic steady-state response of the uniform beam.

The model for forced vibration of Timoshenko beam with discontinuities

In practice, computing harmonic response of the beam, we encounter concentrated loading, concentrated supports, hinges connecting beam segments, concentrated masses or concentrated moments of inertia situated between ends of the beam causing jump discontinuities.

We can apply the Schwartz-Sobolev theory of distributions [3] in order to express jump discontinuities in a quantity, which is to be differentiated. The first-order distributional derivative of a function with a jump discontinuity contains a continuous part and a distributional one which is the product of a magnitude of the jump and the Dirac-delta distribution moved to the point of the jump.

The right-hand side of Eq. 1 is the distributional derivative of the shear force $Q(x,t)$ with $n_1+n_2+n_3$ jump discontinuities. The beam may be supported also between its ends at n_1 concentrated supports with reaction forces of r_i, may carry n_2 concentrated masses of m_i, and may also be subjected to n_3 concentrated transverse forces of f_i and to a distributed transverse loading of $f(x,t)$.

The right-hand side of Eq. 2 is the distributional derivative of the bending moment *M(x,t)* with n_4+n_5 jump discontinuities. The beam may carry n_4 concentrated mass moments of inertia of j_i, and may be subjected to n_5 concentrated force pairs of s_i situated between ends of the beam.

The right-hand side of Eq. 3 is the distributional derivative of the cross-section rotation *φ(x,t)* with n_6 jump discontinuities of ψ_i. The beam may contain n_6 internal hinges connecting beam segments.

The right-hand side of Eq. 4 is the classical derivative of the deflection *w(x,t)* covering the effects of bending and shear deformations [2].

$$\frac{\partial Q}{\partial x} = \rho A(x)\frac{\partial^2 w}{\partial t^2} + \sum_{i=1}^{n_1} r_i\delta(x-\xi_i) + \sum_{i=1}^{n_2} m_i\frac{\partial^2 w(x=\eta_i)}{\partial t^2}\delta(x-\eta_i) - \sum_{i=1}^{n_3} f_i\delta(x-\zeta_i) - f(x,t) \tag{1}$$

$$\frac{\partial M}{\partial x} = Q - \rho J(x)\frac{\partial^2 \varphi}{\partial t^2} - \sum_{i=1}^{n_4} j_i\frac{\partial^2 \varphi(x=\gamma_i)}{\partial t^2}\delta(x-\gamma_i) + \sum_{i=1}^{n_5} s_i\delta(x-\varepsilon_i)\ , \tag{2}$$

$$\frac{\partial \varphi}{\partial x} = -\frac{M}{EJ(x)} + \sum_{i=1}^{n_6}\psi_i\delta(x-\lambda_i)\ , \tag{3}$$

$$\frac{\partial w}{\partial x} = \varphi + \frac{Q}{kGA(x)} \tag{4}$$

Conclusions

Having applied distributions, we present Eqs. 1 to 4 for forced vibration of the Timoshenko beam implying jump discontinuities in the shear force, the bending moment and the cross-section rotation. An analytic closed-form solution to the system of Eqs. 1 to 4 for harmonic steady-state response of the uniform beam is found with integration constants in the form of initial parameters. Making use of this approach, we can find exact expression for the harmonic steady-state response of the beam without summing infinite series and without doing a modal analysis.

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