# **Stochastic Wang Tiles Generation Using the Discrete Element Method**

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**Abstract:** An algorithm for generation of 2D and 3D stochastic Wang tiles is presented in this contribution. The algorithm is based on the discrete element method (DEM) and is therefore applicable for matrix-based structures with separate inclusions. Moreover, the approach is designed for dense packings. An open-source free DEM code YADE was used for all the computations. The method is illustrated on a 2D exemplary realization of Wang tiling and its statistical comparison with periodic tiling.

#### Introduction

In comparison to the up-to-date scheme based on Periodic Unit Cell (PUC), stochastic Wang tilings bring wider diversity in microstructure representation. This concept allows to represent non-periodic material microstructure using limited number of basic tiles [1]. The tiles are boundary-compatible such that during the tiling process, i.e. consecutive placing the tiles side by side to cover up a portion of a plane, there always exist at least two possible tiles to place with boundaries compatible with the previously placed tiles. The random choice results in stochastic realizations of the compressed microstructure.

An algorithm for a design of Wang tiles morphology based on the discrete element method (DEM) [2] is presented in this paper. The algorithm is based on periodic contact detection and proper determination of particles interaction. The method is applicable for representation and reconstruction of random inclusion-matrix microstructures. Apart from the microstructure generation, the approach can also be used to prepare initial positions of particles for DEM itself. This step of DEM simulations is very important in certain applications and might take significant amount of time [3]. With the tiling concept, DEM sample of arbitrary size can be prepared (almost) instantaneously once the tiles are pre-generated. The presented algorithm results in random isotropic dense packings.



Fig. 1: Illustration of the method (left) and resulting tiling (right)

## Algorithm

The approach will be described in detail with an illustrative example (see Fig. 1). For the sake of brevity, 2D case will be presented. However, the concept can be straightforwardly extended to 3D as well as to incorporate different particle shapes. Some implementation details and advantages of free open source YADE software [4] will be addressed.

#### Results

Two applications, the microstructure generation and a simple DEM simulation, will be outlined together with advantages of the approach (in terms of simple statistical descriptors). Fig. 2 shows comparison between periodic tiling (PUC) and Wang tiling in terms of two point probability function.



Fig. 2: Comparison of PUC (left) and Wang tiling (right)

## References

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