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Numerical Simulation of Interaction of Fluid Flow and Elastic Structure Modelling Vocal Fold

Jan Valášek[1,a], Petr Sváček[1,b], Jaromír Horáček[2,c]

[1]Department of Technical Mathematics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Karlovo nám. 13, Praha 2, 121 35, Czech Republic

[2]Institute of Thermomechanics, Czech Academy of Sciences, Dolejškova 5, 182 00 Praha 8, CZ

[a]Jan.Valasek1@fs.cvut.cz, [b]Petr.Svacek@fs.cvut.cz, [c]Jaromirh@it.cas.cz

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Abstract: This paper deals with an interaction between the fluid flow and an elastic body. A simplified model of the human vocal fold is considered. In order to capture deformation of the elastic body the arbitrary Lagrangian-Euler method (ALE) is used. The viscous incompressible fluid flow and linear elasticity models are considered. The problem is solved by the developed finite element method (FEM) based solver. Particularly, for the flow approximation the crossgrid elements are used, whereas for the elastic structure the piecewise linear elements are employed. Results of numerical experiments are shown.

Mathematical model

For the sake of simplicity our study is restricted to a 2D model problem. A scheme of the used model is shown in Figure 1, where Ω^s_{ref} is the reference representation of the structure and Ω^f_{ref} denotes the domain of the fluid.

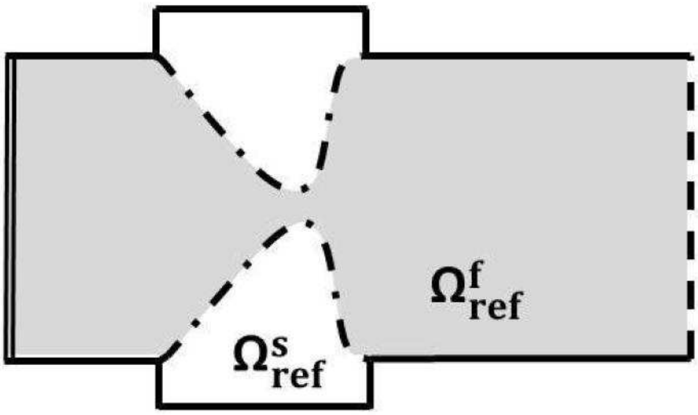

Fig. 1: Schema of vocal folds model in undistorted shape.

The deformation of the structure is then described by the equations (see [1])

$$\rho^s \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\partial \tau^s_{ij}(\mathbf{u})}{\partial x_j} = \mathbf{f}^s \quad \text{in } \Omega^s_{ref} \times (0, \mathrm{T}), \tag{1}$$

where ρ^s is the structure density, $\mathbf{u}$ is the sought deformation, τ^s_{ij} denotes the Cauchy stress tensor and the vector $\mathbf{f}^s$ is the volume density of an acting force. Under the assumption of small displacements and with the help of Hook's law the stress tensor τ^s_{ij} of an isotropic body can be expressed as

$$\tau^s_{ij} = \lambda^s \mathrm{div}\, \mathbf{u} + 2\mu^s e^s_{ij}, \tag{2}$$

where λ^s, μ^s are Lame constants and $e^s_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$.

The motion of the viscous incompressible fluid in the time dependent domain Ω_t^f is modelled by the Navier-Stokes equations in the ALE form (see [2], [3])

$$\frac{D^A \mathbf{v}}{Dt} + ((\mathbf{v} - \mathbf{w}_D) \cdot \nabla)\mathbf{v} - \nu^f \Delta \mathbf{v} + \nabla p = \mathbf{g}^f, \qquad \text{div}\, \mathbf{v} = 0 \quad \text{in}\ \Omega_t^f, \tag{3}$$

where $\mathbf{v}$ is vector of the fluid velocity and p denotes the kinematic pressure, $\mathbf{w}_D$ is the domain velocity of the deformation defined by the ALE mapping, ν^f is the kinematic viscosity of the fluid and $\mathbf{g}^f$ is vector of volume forces. Eq. 1 and 3 are considered with respective initial and boundary conditions, see article [4].

The flow and structure models are coupled with the aid of boundary conditions prescribed on the common interface $\Gamma_{\mathrm{W_t}}$. First, the kinematic boundary condition reads

$$v(x,t) = w_D(x,t) \quad \text{for}\ x \in \Gamma_{\mathrm{W_t}}. \tag{4}$$

Further, the dynamic boundary condition is given by

$$\sum_{j=1}^{2} \tau_{ij}^s(X)\, n_j^s(X) = \sum_{j=1}^{2} \sigma_{ij}^f(x)\, n_j^s(x), \quad i = 1,2, \quad x \in \Gamma_{\mathrm{W_t}},\ X \in \Gamma_{\mathrm{W_{ref}}}, \tag{5}$$

where σ_{ij}^f is the stress tensor of fluid, $n_j^s(X)$ denotes the components of the unit outward normal to the interface $\Gamma_{\mathrm{W_{t=0}}}$ pointing into Ω_{ref}^f.

Numerical approximation

The both model equations are discretized with constant time step. In the case of structure the space discretization by FEM piecewise linear elements are performed and then the system of ordinary differential equations of second order is approximated by the Newmark method.

On the other hand Eq. 3 is discretized in time with help of BDF2 scheme and then for space discretization so called cross-grid P1 elements are used. The acquired nonlinear equations are lineared by Oseen linearization and iterative solution is seeked with the support of mathematical library UMFPACK.

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