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Computer-Aided Plastic Limit Analysis of Plates

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Abstract: This paper aims to explore the potential of using the yield line theory for design of reinforced concrete plates. The primary focus is development of a computer program able to solve arbitrary yield line systems. The program will include a GUI, which will allow for quick and intuitive input and analysis of yield line systems regardless of the complexity of their analytical solution. Furthermore, optimization of orthotropic reinforcement will be implemented. Possibility of use of the program for design compliant with Eurocode 2 will be discussed.

Introduction

It has been shown e.g. in [1, 2] that design of reinforced concrete plates based on plastic limit analysis (specifically the yield-line theory) can lead to a significant reduction in reinforcement volume as opposed to the elastic approach mostly employed in engineering practice. The yield-line theory gives an upper bound of the limit load and it is therefore important to find the right collapse mechanism which gives the lowest limit load. The solution is also influenced by the ultimate bending moment, which for orthotropically reinforced concrete plates can depend on the angle between a plate cross section (i.e. a yield-line) and the main reinforcement direction.

Optimizing collapse mechanisms has been extensively covered in [3, 4], but optimization of reinforcement proportion[1] has not received much attention. This paper aims at providing a user friendly computer program for the analysis of yield-line mechanisms, allowing both geometry and reinforcement optimization.

Both [1, 2] offer an extensive validation of computations obtained by the yield-line theory by experiments. It is always assumed, however, that the reinforcement ductility is sufficient due to the plates being lightly reinforced. Therefore we investigate whether some still feasible reinforcement ratios and proportions can occur for which the optimal collapse mechanism is not able to develop.

The program

The program was developed in Python 2.7 environment using the Enthought TraitsUI package for the graphical interface. A general yield-line mechanism is defined through so-called 'plate segments' (henceforth refered to as 'segments'), which in turn consist of nodes. The whole object structure is evident from the tree view in Fig.1. The segments represent the rigid, mutually rotated parts of a yield-line mechanism. A node in general has three coordinates (two coordinates in plane and one deflection coordinate) of which some may be unknown. A well-defined mechanism must be unambiguous (must not be under-determined) and must be geometrically compatible (must not be over-determined). The basic unknowns of every mechanism are three parameters per segment and with more complex mechanisms some nodal coordinates may have to be unknowns as well in order to assure the geometrical compatibility.

[1]Assuming a plate reinforced in two mutually perpendicular directions, we will refer to the ratio of reinforcement area to concrete area in both directions as reinforcement ratio, as opposed to the ratio of the two reinforcement areas, which we will call reinforcement proportion.

Input data specific to optimization are managed by the Parametric Study object. There are two categories - node data specify node coordinates and plate data specify plate attributes (in practice mostly reinforcement ratios, but any attribute may be chosen). Both data types then specify an optimization parameter on which they depend. That enables the user to set the conditions precisely and also save computation time, as more values can depend on one parameter. Optimization parameters are kept separately and may have their bounds specified. The limit load must be minimal according to node positions but at the same time maximal according to plate attributes. The optimization itself is performed by the SciPy package optimize library - two function calls, one encapsulated in the other, are used to fulfil both the minimizing and the maximizing criteria.

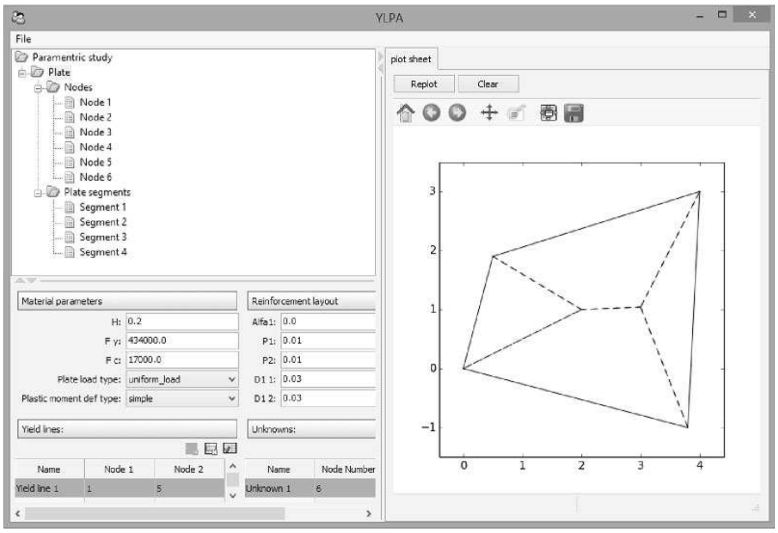

Fig. 1: The user interface

Ductility of collapse mechanisms

As has been stated, we want to go beyond the simple assumption of lightly reinforced plates. Our program contains a routine for checking rotational capacity across all yield-lines. That allows us to easily discover cases where a seemingly optimal collapse mechanism cannot develop due to insufficient ductility. As it is unclear how to define a moment-curvature relation in a general cross section of a plate, we work with an equivalent reinforcement ratio based on the angle of a section with the main reinforcement direction. For concrete, the stress-strain law for nonlinear calculations according to [6] has been utilized.

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