21st International Conference **ENGINEERING MECHANICS 2015**
Svratka, Czech Republic, May 11 –14, 2015
Full Text Paper **#200**, **pp. 37–43**

Statistical evaluation of three and four-point bending tests of FRC

Adam Podstawka[1, a *], Marek Foglar[1,b] and Vladimir Kristek[1,c]

[1] CTU in Prague, Faculty of civil engineering, Department of concrete and masonry structures, Thákurova 7, 166 29, Prague, Czech Republic, Tel: +420 224 354 630

[a]adam.podstawka@fsv.cvut.cz, [b]marek.foglar@fsv.cvut.cz, [c]kristek@fsv.cvut.cz

Keywords: Fiber reinforced concrete, testing of materials properties, force-deflection diagram, numerical modeling, statistical evaluation

Abstract. Paper compares and discusses three-point bending and four-point bending tests of fiber reinforced beams. A numerical modeling, stochastic and statistical analysis are used to determine which layouts of testing are more suitable for identification of mechanical and fracture properties of fiber reinforced concrete.

Introduction

Mechanical properties of fiber reinforced concrete (FRC) are commonly tested on the beam-shaped specimens loaded by bending moment and by the corresponding shear force.

The mechanical approach according to the RILEM and Eurocode [5] recommendations, use the three-point bending test of notched specimens. Dimensions of specimen are 150x150x550mm. Depth of the notch is 25mm. Presence of the notch clearly predefines the position of the initial crack which generally is not able to respect the actual fiber distribution.

When the engineering approach according to Czech and German standards [2, 3] is used, the fracture characteristics of FRC are tested using the four-point bending tests. The specimens are beams, 150x150x700mm, with the span of 600mm. No notch is used for the predefinition of the position of the macro-crack. The specimen is loaded by four-point bending. The loading forces divide the span into thirds. At the specimens without the notch, the macro-crack propagates at the weakest cross-section. Layout of both types of the experiments can be seen in Figure 1.

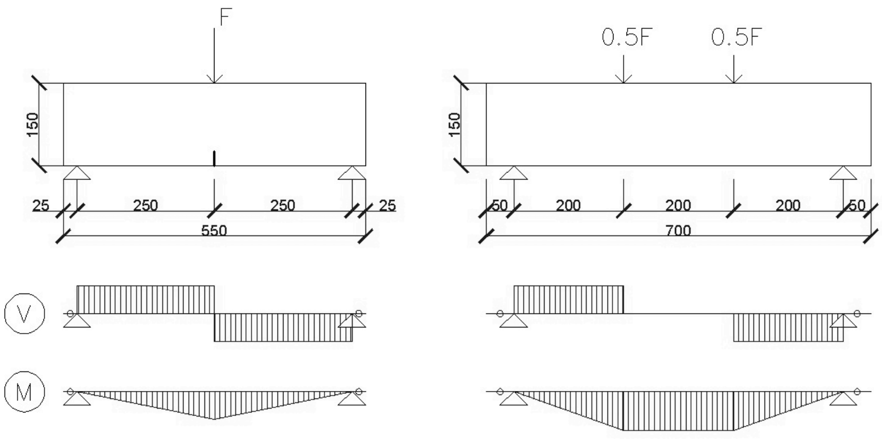

Figure 1 Layout of the experiments (on the left is three-point bending test)

The benefit of the four-point bending test arrangement is the constant value of the bending moment with no shear force in the middle third of the specimen and thusly elimination of the effect of shear force. At the specimens without the notch, the macro-crack propagates at the weakest cross-section, i.e. at the cross-section with the smallest fiber content or with unsuitable fiber distribution. On hand of a simple testing example, the paper compares and discusses two different layouts of testing of mechanical and fracture properties of FRC with respect to the scatter of obtained results.

The main assumption was that presence of notch (in case of three point bending test) may cause that initial crack can appear in a non-representative cross-section without respect to average material characteristics throughout the specimen. On the other side, in the case of un-notched specimen, the initial crack propagates in the weakest cross-section, i.e. in the cross-section with the lowest tensile strength with the smallest amount of fibers.

Comparison methods

To confirm or refute assumptions above and for decision which method is more suitable for identification of material characteristic of fiber reinforced concrete, three methods were used: method of tensile chord, method of direct evidence and method of dispute.

Method of tensile chord. This method [7] is quite simple, but very effective. On the example of a tensile chord it is shown that due to lower statistical sensitivity, the four-point bending test without notch is more suitable for testing of material characteristics of FRC specimens.

In case of un-notched specimen, fracture propagates in the weakest cross-section, i.e. in the cross-section with the lowest tensile strength with the smallest amount of fibers. The mentioned phenomenon is tested in following application.

In this method, concrete strength class represents the mean value of flexural strength f_{ctm} the value of strength of 100%. The lower quantile of flexural strength $f_{ctk,05}$ represents 69% of mean value f_{ctm} and the upper quantile $f_{ctk,95}$ represents 131% of mean value f_{ctm} for typical strength class of concrete. According to Gaussian curve, the distribution of concrete strength lies in mentioned borders 69% and 131% of mean value.

Tension test of specimen was used for simulation and statistical approach of strength parameters. Un-notched specimen is divided into ten elements, where crack propagation is possible. The random strength (from 69% to 131% of f_{ctm}) was generated for every element and the strength of the whole specimen is represented by strength of the weakest element. On the contrary, the notched specimen is the only one element, where crack propagation is controlled. The random strength (from 69% to 131%) is assigned for this weakest element which represents the strength of the whole specimen. Both options are represented by five specimens. Twenty simulations were made in total. Layout of the tests can be seen in Figures 2 and 3.

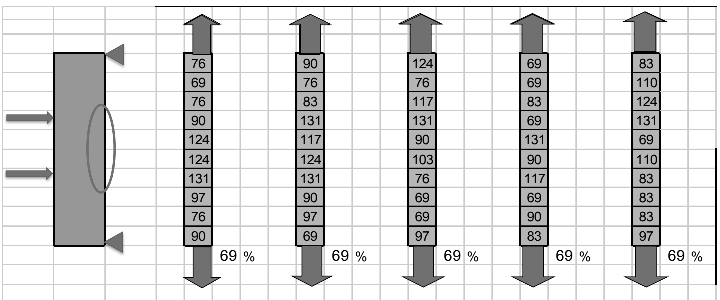

Figure 2 Un-notched specimen simulations

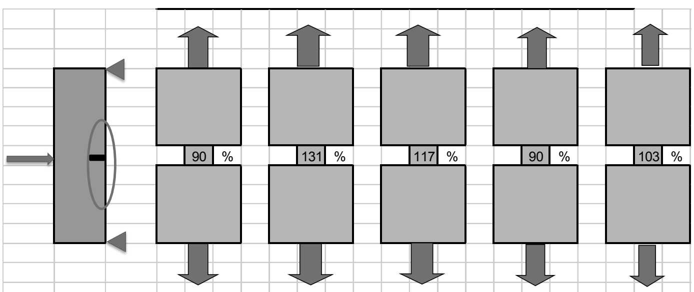

Figure 3 Notched specimen simulations

The value of tensile strength was randomly generated and then statistically evaluated. Compared values were average strength, variability, standard deviation and coefficient.
For the randomly generated dataset, the average flexural strength which determines the critical force for crack propagation is less in every simulation for four-point bending tests without notch than for three-point bending tests with notched specimens.
The standard deviation in the set of five specimens is under 7% of mean value of concrete flexural strength for four-point bending test with un-notched specimens and about 20% of mean value for three-point bending tests on notched specimens.
Also coefficient of variation is lower for four-point bending test with un-notched specimen in every simulation.
Twenty simulations of randomly assigned values of concrete strength were made to determine the statistical scatter of results of bending tests. Results of the simulations showed bigger statistical sensitivity of flexural strength determined by three-point bending test on notched specimen with one element. On the opposite side lies four-point bending test on un-notched specimen with very low statistical sensitivity of flexural strength determined in possible damage zone composed by ten elements.
Due to lower statistical sensitivity, four-point bending test on un-notched specimen is more suitable for testing FRC specimens to determine flexural strength of material.

Method of direct evidence. More sophisticated method of direct evidence follows up the method of tensile chord and confirms the greater statistical sensitivity of three-point bending test arrangement. The numerical simulations and probabilistic approach were used in this method.
The primary assumption of the numerical approach was to perform the numerical analysis of FRC based on previously published approaches [1, 4, 8, 9] and perform a study of transferability of these approaches and development of own application routines.
For numerical simulation of experiments, the commercial program ATENA 2D was used. A numerical model was created with macro-elements which represent beam-shaped specimen and load distributed by steel plates. Final numerical models including meshes for both types of bending tests are shown in Figure 4.

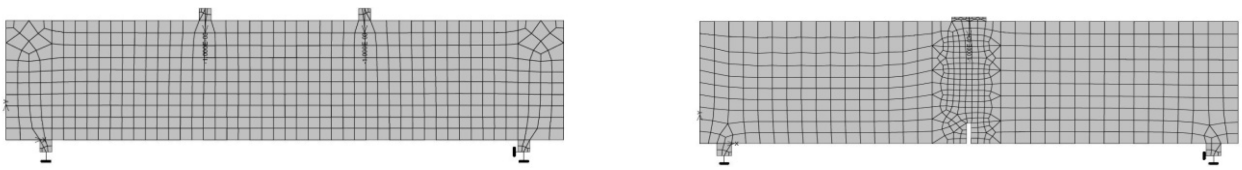

Figure 4 Numerical models, mesh arrangements

The material model 3DNLC2U was used for the non-linear analysis. Only this material model 3DNLC2U can appropriately describes real behavior of FRC including its characteristic peak and following hardening or softening (see Fig. 5). The agreement between the experiments and the simulations is very good.

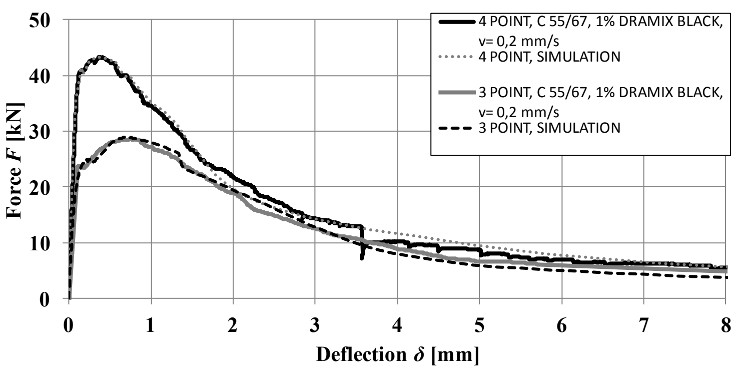

Figure 5 Comparison of numerical simulations and experiments

The commercial program SARA was used for probabilistic assessment. The probabilistic models of selected mechanical properties (shown in Table 1) could be chosen according to the recommendation of JCSS [6]. The value of fracture energy is not a parameter of the material model 3DNLC2U. It has to be determined how to vary fiber amount. A possible approach is to change the position of the point on the tension-softening curve.

Table 1 Probability density functions of selected parameters

Parameter	**Des.**	**Unit**	**PDF**	μ	V
Tensile strength	f_t	MPa	LN 2.par	f_t	0.25
Point on ten-soft curve	*a*	-	LN 2.par	Middle of the interval	0.30

Hundred numerical simulations were made for both types of bending tests. The results are the generated force-deflection diagrams. The areas under these force-deflection diagrams were statistically evaluated (see Table 2). The compared values were average, standard deviation and coefficient of variation.

Table 2 Results of statistical evaluation

	ϕ **[kN.mm]**	S **[kN.mm]**	V **[-]**
3Point	0.659	0.204	0.309
4Point	0.577	0.167	0.289

Even if for the randomly generated dataset, the value of average, standard deviation and coefficient of variation is less in every simulation for four-point bending tests without notch, it was demonstrated that random distribution of selected material model parameters cannot be used for the whole specimen.

The selected material parameters are not mechanical properties of real material, but only parameters of material model. There is no generation of random simulations of experiment, but only editing behavior of numerical model. Therefore, an inverse approach must be chosen, this approach was called the method of dispute.

Method of dispute. This method combines knowledge obtained from the method of tensile chord and the method of direct evidence. The main idea was that the mechanical properties of the weakest cross section determine mechanical properties of whole specimen. For both types of bending test the typical weakest cross-sections in the damage zone were chosen.

In the non-linear analysis the material model 3DNLC2U was again used. It was necessary to create uniform mesh for the probabilistic simulation to be performed in the following step. The element size was chosen 5mm for three-point bending test and 15mm for four-point bending test in the damage zone (zone in the middle third of span of four-point bending test). The used meshes are shown in Figure 6.

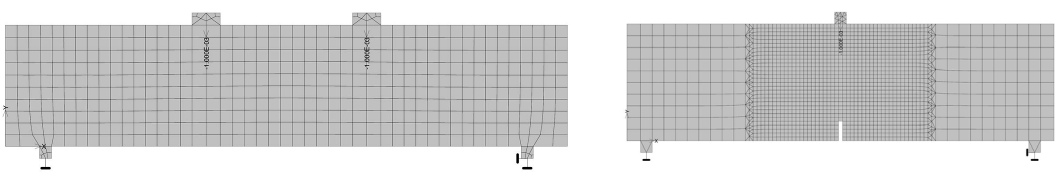

Figure 6 Numerical models, mesh arrangements

The program SARA was used for probabilistic assessment. The selected mechanical properties and its assigned probabilistic models are shown in Table 3.

Table 3 Probability density functions of selected parameters

Parameter	**Des.**	**Unit**	**PDF**	**μ**	**V**
Tensile strength	f_t	MPa	LN 2.par	f_t	0.25
Point on ten-soft curve	a	-	Rectangular	a=0.25	b=0.8

The main idea was that the mechanical properties of the weakest cross section determine mechanical properties of whole specimen. For both types of bending test the typical weakest cross-sections in the damage zone were chosen. These weakened cross-sections must be located around the notch in case of three-point bending test, because bending moment is reduced with the increasing distance. Considered sections are in 5 (S05), 15 (S15), 30 (S30) a 45 (S45) mm from the notch. The arbitrary position is in the middle third of span in the case of four-point bending test. The selected parameters of the material model of these weakened sections were assigned to probabilistic models according [6].
Hundred numerical simulations were made in total for both types of bending tests (20 for each cross-section). The results are the generated force-deflection diagrams. Twenty simulations were made for each section, but only four-point bending test registered presence of weakened cross section in each simulation. On the opposite side, only a few random simulations registered presence of weakened in case of three-point bending tests (see Figure 7).

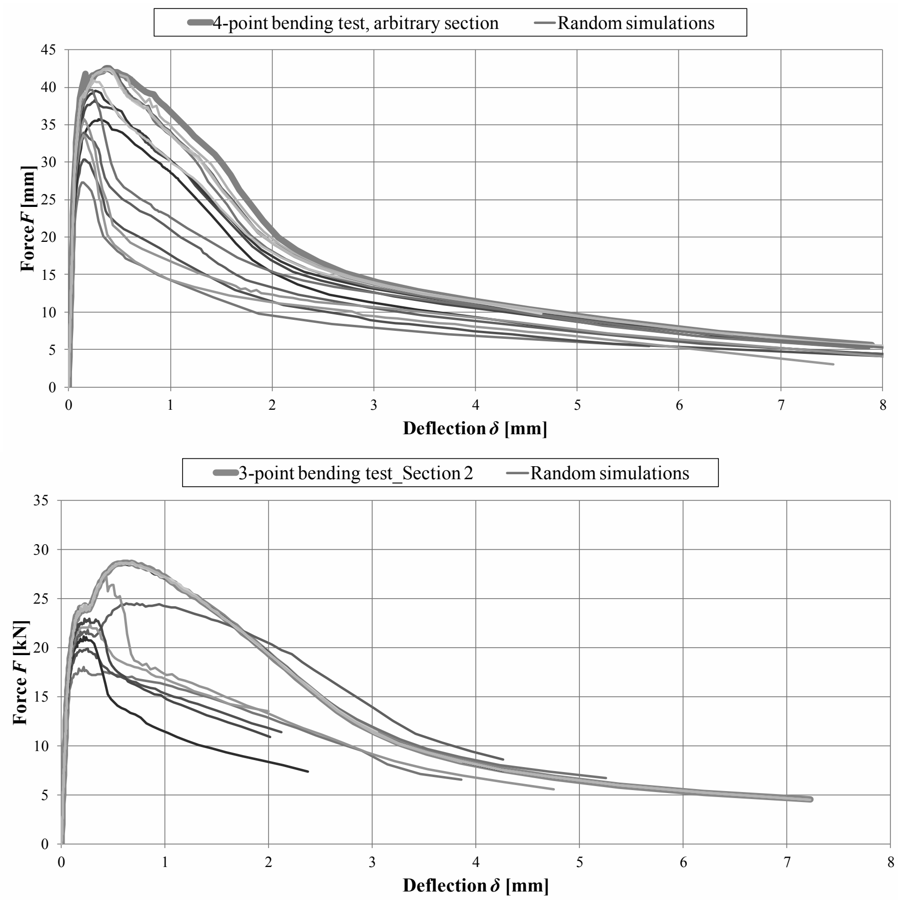

Figure 7 Random simulations (4-point on the left, 3-poin on the right)

The areas under the force-deflection diagrams were statistically evaluated. The compared values were average (ϕ), standard deviation (S), coefficient of variation (V), skewness (α) and spikiness (ε). The results of statistical evaluation can be seen in Table 4 (S means section).

Table 4 Results of statistical evaluation

	ϕ	S	V	α	ε
4-point	0.834	0.180	0.216	-0.965	-0.657
3-point, S05mm	0.869	0.182	0.210	-1.266	0.608
3-point, S15mm	0.887	0.169	0.190	-1.105	-0.483
3-point, S30mm	0.980	0.047	0.048	-2.501	5.489
3-point, S45mm	1.000	0.000	0.000	-	-

For the randomly generated dataset of the three-point bending tests with a notch, the value of the average is increasing with the increasing section distance.
Values of standard deviation are comparable for both types of tests only for very low distance (5 mm) between weakened section and notch in case of three-point bending test.
On the other hand coefficient of variation is decreasing with the increasing section distance. The values of spikiness and skewness are not relevant but increasing cumulation around the average for three-point bending tests that can be observed.
The results confirmed that three-point bending test cannot locate the weakest cross section (cross-section with the lowest tensile strength with the smallest amount of fibers). On the contrary, the four-point bending test always locates the weak cross section (in the middle third of span).

Summary

The main purpose of this paper was to compare the commonly used bending tests arrangements and determine which one is more suitable for testing of mechanical and fracture properties of FRC.

It was determined that three-point bending test arrangement with the notch is not able to locate the weakest cross-section, which is absolutely essential for non-homogeneous material, such as FRC. The results are always influenced by the material constitution profile over the notch, which does not correspond to random distribution of tensile strength and fiber content in beam.

The four-point bending test can safely localize the weakest (representative) cross-section, which is basic assumption to obtain representative results.

Moreover, four-point bending test investigates about 40-200 times larger area of possible rise of the crack (depending of width of the notch). This fact rapidly increases the probability of localization of the weakest cross section.

Based on the above assumptions it is possible to argue that four-point bending test is only suitable for identification of the material parameters of FRC.

Acknowledgements

Support from the Grant Agency of the Czech Republic, Grant Project No. 13-30441S, is gratefully acknowledged.

References

[1] M. Kovář, M. Foglar, Analytické vyjádření mechanických a lomových vlastností vláknobetonu, In 20, Betonářské dny 2013, Česká betonářská společnost ČSSI, Praha, 2013, 258-263.
[2] TP FC 1-1, Technické podmínky 1: Vláknobeton - Část 1: Zkoušení vláknobetonu, Českomoravský beton, a.s., Praha, 2007, p. 34.
[3] DAfStb-Richtlinie Stahlfaserbeton, Deutscher Ausschuss Für Stahlbeton, Berlin, 2010, p. 52.
[4] Kovar M, Foglar M., An analytical description of the force–deflection diagram of FRC. Composites: Part B (2014), http://dx.doi.org/10.1016/j.compositesb.2014.10.021

[5] RILEM TC 162-TDF, Test and design method for steel fibre reinforced concrete, Bending test, Final recommendation, Materials and Structures (2002), 35(253):579–82.
[6] JCSS probabilistic model code part 3: Resistance models, 3.01Concrete properties, 2002. http://www.jcss.byg.dtu.dk/Publications/Probabilistic_Model_Code
[7] M. Kovar, V. Kristek, M. Foglar, A. Podstawka, Three or four-point bending tests for identification of material characteristics of fiber reinforced concrete, Proceedings of 10th CCC Congres Liberec 2014, Czech Concrete Society, Praha, 2014.
[8] M. Kovář, M. Foglar, P. Veselý, Vliv rychlosti zatěžování vláknobetonu na jeho pevnostní a lomové charakteristiky, Konstrukce (2013), 36-39.
[9] M. Foglar, M. Kovar, Conclusion from experimental testing of blast resistance of FRC and RC bridge decks, International Journal of Impact Engineering. 59 (2013), 18-28.