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DUCTILE FRACTURE PATTERN UNDER FATIGUE CONDITIONS

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Abstract: *Cyclic plasticity lies at the root of fatigue fracture in the ductile substances. Concurrently, plastic material models solving the total loading path are applying to computer processing very exacting, on the high numbers of load cycles. Consequently, for that end, in the treatise presented is introduced, in line with (Seiler, 2020), a model with especially small computational labor. It yields a macroscopic characterization of fatigue fracture by mixing the phase-field technique for brittle fracture with typical resistance interpretation. We obtain a local lifetime variable that depreciates the fracture durability gradually. When inferring the stress-strain path at the most, from cyclic material characteristic, only one increment per load cycle is needed. The version permits to depict fatigue crack initiation, growth and residual fracture.*

Keywords: Degradation, Elasto-plastic stress, Fracture, Phase-field, Propagation.

1. Introduction

Crack initiation is defined as the aspect of a crack of several millimeters. At the same time, statistical long life interpretations based on Wöhler tests are used. Wöhler curves depict the number of load cycles until crack initiation in view of fact a certain nominal stress amplitude. Except that from strain-controlled background it is possible to deduce the cyclic behavior of the material itself. The strain Wöhler curves are eg made use of within the local strain way of tackling that is numbered among the advanced durability interpretations efficient to consider the effect of plastic strain.

In the case of optimized construction, the crack growth has to be contemplated during life prediction, too, in order to design the structures frugally. Concurrently, the damage tolerant approach is acquired. Goal magnitude is the number of cycles it takes a crack to propagate from an exactly visibly detectable extend to failure. Fracture mechanics yields tools to depict this crack growth as for instance the stress intensity factor.

Some authors deduce fracture mechanics with local strain approach, inferring crack driving force for fatigue crack growth. Instead of a sharp crack that is an elastic arrangement causes a stress singularity, they contemplate plasticity at the crack tip that is included by equivalent residual stresses.

In the treatise presented is introduced, in line with Seiler at al., a model giving rise to relatively small computational labor, because an elasto-plastic material evaluation.

In addition to statistical and analytical methods, crack growth can be simulated numerically. Whereas the simulation of developing cracks applying a sharp crack topology displays difficult particularly in 3D settings, disperse representations of the crack path may yield significant benefits from a computational stand point.

Lately, several suggestions to extend the phase-field technique to fatigue have been published. Even though the authors employ diverse approaches, they are not yet getting over one key demanding task belong to fatigue crack initiation and growth. The enormous computational labor regards to the high number of load cycles.

The line-up of the phase-field method with the local strain approach is substantially more flexible than the merely LSA, which itself is not applicable to crack growth by no means.

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2. Phase-field representation for fatigue fracture

This model for brittle fracture reading is based on the Griffith criterion of linear elastic theory. It suggests that a brittle crack can solely grow if the fracture energy, that is released in the course of the thing formed of new crack surface, equal to the critical energy release rate or fracture toughness G_c. Employing a variation formulation for the criterion mentioned, namely by minimizing the total energy regarding the displacement field and crack geometry, random crack paths likewise as crack initiation can be simulated without any further criteria.

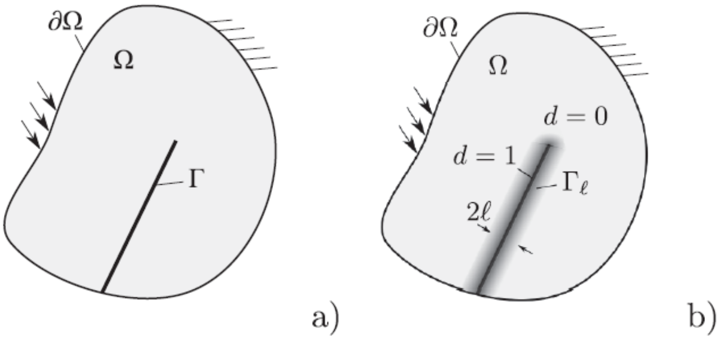

Fig. 1: Fracture region Ω with crack surface Γ:a) Sharp example of crack topology; b) arranged example. The crack is described by the phase-field variable d = 1, whereas d = 0 means undamaged material. The crack is arranged over the length scale l.

The total potential Π for a region Ω comprising a fracture surface Γ, according to Fig.1a, can be expressed as

$$\Pi = \int_\Omega \psi^e(\boldsymbol{\varepsilon})\,\mathrm{d}V + \int_\Gamma \mathcal{G}_c\,\mathrm{d}A, \tag{1}$$

Supposing linear elasticity and small strains ε, the elastic strain energy density can be rendered as $\psi^e = \frac{1}{2}\lambda\,\mathrm{tr}^2(\boldsymbol{\varepsilon}) + \mu\,\mathrm{tr}(\boldsymbol{\varepsilon}^2)$ with the elastic constants λ and μ.

On the score for a useful numerical implementation a regularization is proposed. To depict the crack topology, an additional field variable d [0, 1] is inserted smoothly bridging the entirely intact (d = 0) and totally broken (d = 1) situation. The material states were interpreted as phases and the term phase-field was introduced. After approximating the sharp crack by a crack density γ_c being subject on a length scale parameter l (Fig. 1b), the regularized energy functional may be related in the form

$$\Pi_\ell = \int_\Omega g(d)\psi^e(\boldsymbol{\varepsilon})\,\mathrm{d}V + \int_\Omega \mathcal{G}_c \underbrace{\frac{1}{2\ell}\left(d^2 + \ell^2\left|\nabla d\right|^2\right)}_{\gamma_\ell}\mathrm{d}V. \tag{2}$$

In conformity with damage mechanics, a degradation function $g(d) = (1 - d)^2$ is brought in that simulates the loss of stiffness owing to the evolving crack. Moreover, it gets together the mechanical field **u** and face-field **d**. The stress reads

$$\boldsymbol{\sigma} = g(d)\frac{\partial\psi^e}{\partial\boldsymbol{\varepsilon}}. \tag{3}$$

From the variation $\delta\Pi_l = 0$ we can deduce the governing equation

$$\vec{0} = \mathrm{div}\boldsymbol{\sigma} \qquad d - \ell^2\Delta d = \left(1 - d\right)\underbrace{\frac{2\ell}{\mathcal{G}_c}\psi^e(\boldsymbol{\varepsilon})}_{\mathcal{H}} \tag{4}$$

Following out the boundary conditions $\boldsymbol{n}\cdot\boldsymbol{\sigma} = \tilde{\boldsymbol{t}}$, $\boldsymbol{u} = \tilde{\boldsymbol{u}}$ and $\boldsymbol{n}\cdot\nabla d = 0$ with $\boldsymbol{t}^{\sim}$ and $\tilde{\boldsymbol{u}}$ being the decreed boundary tractions and displacements. In Eq. (4), the crack driving force $\hat{H}$, that regularizes the evolution of the phase-field, may be recognized. Further, the face-field variable was interpreted in the sense of damage and thus the crack driving force as the maximum energy density in the course of time.

$$\mathcal{H} = \frac{2\ell}{\mathcal{G}_c} \max_{s\in[0;t]} \psi^e\left(\varepsilon, s\right), \tag{5}$$

Providing local irreversibility for the phase-field, physically motivated, the degradation can thus only be applied to the tensile part of the energy density

$$\psi_l^e(\varepsilon, d) = g(d)\psi_+^e(\varepsilon) + \psi_-^e(\varepsilon), \tag{6}$$

whereas the compressive line remains unaffected. The strain is thus split into a volumetric and a deviatoric part, in conformity with its principal constituents on the crack orientation.

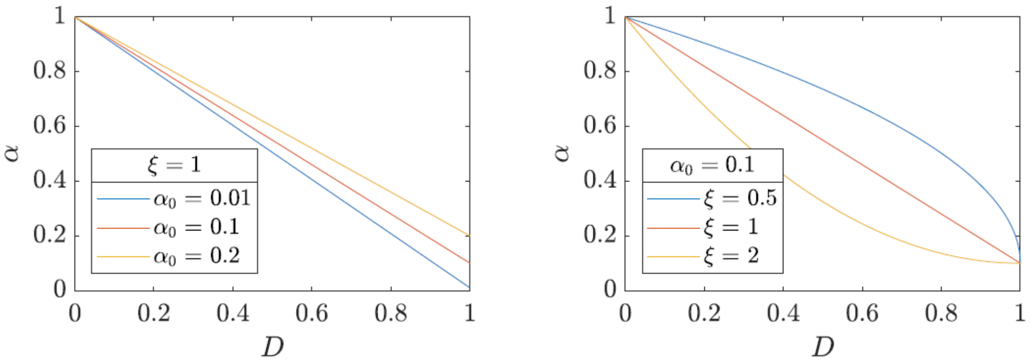

Fig. 2: Fatigue degradation function $\alpha(D) = (1- \alpha_0)(1-D)^{\xi} + \alpha_0$, that depreciates the fracture toughness which depends on the local life variable.

3. Enlargement to fatigue

Similarly, to Carrara, the fracture toughness G_c is decreased if the material degradation owing to repeated stress precedes, or this can be depicted by a local lifetime variable. An additional scalar *fatigue degradation function $\alpha(D)$* from interval $[\alpha_0, 1]$ with $0 < \alpha_0 < 1$ is brought in that puts down the fracture toughness locally. The energy functional is then given

$$\Pi_\ell = \int_\Omega g(d)\psi^e(\varepsilon)\mathrm{d}V + \int_\Omega \alpha(D)\mathcal{G}_c \frac{1}{2\ell}\left(d^2 + \ell^2\left|\nabla d\right|^2\right)\mathrm{d}V \tag{7}$$

The decrease of the total energy owing to α(D) is meant to make a model the dissipation due to local cyclic plasticity. Needed again $\delta\Pi_l = 0$, the evolution equation of the phase-field on factor of the irreversibility conditions (5) extends to

$$\alpha(D)d - \nabla\alpha(D)\cdot\ell^2\nabla d - \alpha(D)\ell^2\Delta d = \left(1 - d\right)\max_{s\in[0;t]} \psi^e(\varepsilon)\frac{2\ell}{\mathcal{G}_c} \tag{8}$$

The lifetime variable $D \sim [0, 1]$ is accumulated absolutely locally, it can be explained as a damage variable of a special linear character: For $D = 0$ a material point has undergone no fatigue loads by no means, whereas $D = 1$, it has experienced all load cycles it can possibly bear before vanishing its integrity, linearly spanning the life in between. By virtue of the preceding contention, the fatigue degradation function

$$\alpha(D) = (1 - \alpha_0)(1 - D)^{\xi} + \alpha_0 \tag{9}$$

For the suggested parameters α_0 and ξ (Fig. 2) For $D = 0$, the material has experienced no cyclic loads at all and therefore must have full fracture toughness, consequently $\alpha(0) = 1$ must hold. The parameter ξ controls the relation between D and d. The threshold α_0, on the other hand, yields a link to experiments on residual fracture in which the remaining fracture toughness of fatigued components is measured. It has to be larger than zero, since even cyclically damaged material at the end of its lifetime offers a certain resistance against crack propagation.

As the phase-field variable is expound as physical damage model instead of as an indication for a potential crack, the irreversibility condition (5) is chosen. But a tension compression split is not necessary.

In respect of the problem tackled it is commendable to warn about these days subjects upheld among specialists, e.g. (i) the non-isothermal non-local framework for the evolution of damage, fatigue and fracture in materials under the hypothesis of small deformation, (ii) fatigue in metallic systems from the perspective of interactions between the microstructure, the deformation mode and the mechanical state of both low and high temperatures, (iii) phase-field models of brittle fracture and a fast hybrid formulation, and (iv) damage and fatigue represented by a functional derivative model.

4. Conclusions

A fatigue life interpretation together with the phase-field technique for fracture is given. In one- and two-dimensional illustrations this applications has already been indicated. After depreciating the fracture toughness that is dependent on a local life variable, the fatigue influences are taken into account. The variable mentioned is specified by the so-called local strain way of tackling, contemplating plasticity as the reason of ductile fatigue fracture. On top of that, several load cycles can be modeled within one increment.

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