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UNCERTAINTIES OF KINEMATIC AND DYNAMIC MODEL OF REDUNDANTLY ACTUATED PARALLEL MECHANISMS

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Abstract: *The paper deals with modeling of redundantly actuated parallel mechanisms. A calculation method how to obtain a quantitative information about kinematic and dynamic model accuracy is presented. This method is based on utilization uncertain model parameters such as geometric and dynamic parameters with bounded deviations. The proposed method is tested on the parallel redundantly actuated planar* $4\underline{R}RR$ *mechanism "Crosshead". Finally obtained results are discussed.*

Keywords: Parallel kinematics, Redundant actuation, Model uncertainties, Forward kinematics.

1. Introduction

The parallel mechanisms have potential to improve stiffness, position accuracy, dynamic and payload over their serial counterparts (Merlet, 2000). These features can be further improved by the redundant actuation (Valasek et al., 2004). Howewer control of the redundantly actuated parallel mechanism suffers from several new control problems like the danger of emerging parasitic feedback forces and mutual fighting of the redundant actuators (Muller, 2010). This is especially problematic if the model imperfections appear. The motivation for this paper is to quantitatively describe the uncertainties of kinematic and dynamic model of the redundantly actuated parallel mechanisms. This is essential knowledge for design of a robust control method for the mechanisms (with uncertain model) and good control performance.

2. Kinematic model with uncertain geometric parameters

The mechanisms based on the parallel kinematic structure consist of a moving platform with an end-effector connected to the base by several kinematic chains. The kinematic chains represent closed kinematic loops, which leads to *r* geometric constraints (model)

$$\mathbf{g}(\mathbf{q},\mathbf{l}) = \mathbf{0}, \tag{1}$$

where $\mathbf{q}$ is a vector of *n* general coordinates and $\mathbf{l}$ is a vector of nominal geometric parameters.

We assume that the real geometric parameters $\underline{\mathbf{l}}$ (mechanism) differ from the nominal parameters $\mathbf{l}$ (model of the mechanism) as follows

$$\underline{\mathbf{l}} = \mathbf{l} + \Delta\mathbf{l}, \tag{2}$$

where $\Delta\mathbf{l}$ are bounded uncertainties of the nominal geometric parameters $\mathbf{l}$. The vector of coordinates $\mathbf{q}$ should be appropriately assigned to the model of the mechanism as $\mathbf{q}$ and to the (real) mechanism as $\underline{\mathbf{q}}$. Now we can write *r* geometric constraints for the mechanism

$$\mathbf{g}\left(\underline{\mathbf{q}},\underline{\mathbf{l}}\right) = \mathbf{0}. \tag{3}$$

Further we assume that only a subset $\underline{\mathbf{q}}_{\mathrm{m}}$ of $\underline{\mathbf{q}}$ is measured and a complement $\underline{\mathbf{q}}_{\mathrm{f}}$ of $\underline{\mathbf{q}}$ (unmeasured) remains unknown. To tackle this problem a computational algorithm was designed. The algorithm is

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based on usage of measured coordinates $\underline{\mathbf{q}}_{\mathrm{m}}$ during calculation of (1), where the nominal geometric parameters $\mathbf{l}$ are extended by their uncertainties $\Delta\mathbf{l}$. The uncertainties $\Delta\mathbf{l}$ are selected in N iterative steps as $\Delta\mathbf{l}_{\mathrm{i}}$ (i = 1, 2, ..., N), where $\Delta\mathbf{l}_{\mathrm{i}}$ is a random selection from the tolerance area. Finally the obtained solutions $\mathbf{q}_{\mathrm{fi}}$ are evaluated and a kinematic model error $\Delta\mathbf{q}_{\mathrm{f}}$ is determined (Fig. 1).

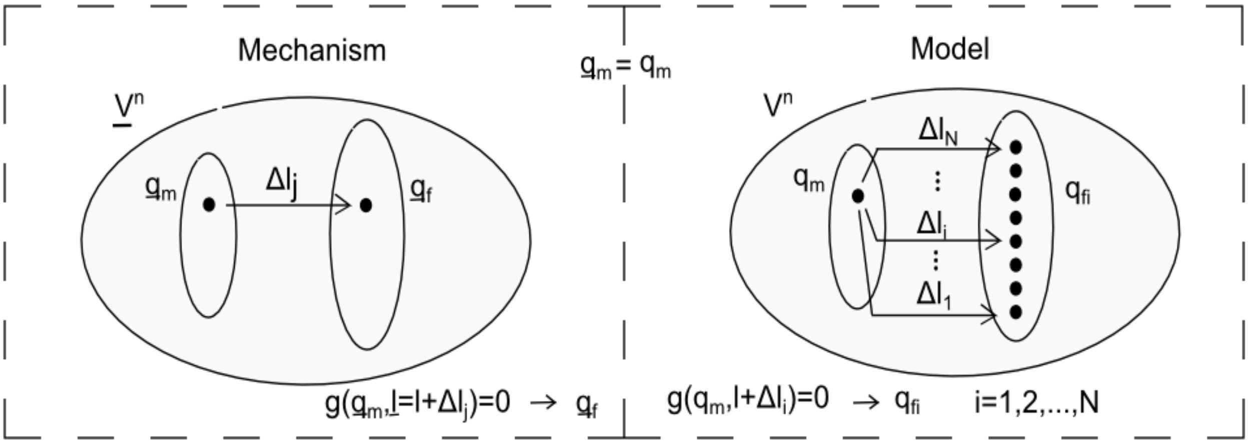

Fig. 1: Kinematic model with geometric imperfections.

3. Dynamic model with uncertain parameters

The dynamic model of redundantly actuated parallel mechanism is represented by δ ordinary differential equations of motion (Stejskal, 1996) for δ independent coordinates (δ degree of freedom of mechanism). The vector of all n coordinates $\mathbf{q}$ is separated into δ independent ones $\mathbf{q}_2$ and n- δ dependent ones $\mathbf{q}_1$

$$\bar{\mathbf{G}}(\mathbf{q},\mathbf{l},\mathbf{m})\ddot{\mathbf{q}}_2 + \bar{\mathbf{C}}(\mathbf{q},\dot{\mathbf{q}},\mathbf{l},\mathbf{m})\dot{\mathbf{q}}_2 + \bar{\mathbf{Q}}(\mathbf{q},\dot{\mathbf{q}},\mathbf{l},\mathbf{m},\mathrm{t}) = \mathbf{A}^T(\mathbf{q},\mathbf{l})\mathbf{u}, \tag{4}$$

where $\bar{\mathbf{G}}$ is a generalized mass matrix, $\bar{\mathbf{C}}\dot{\mathbf{q}}_2$ represents generalized Coriolis and centrifugal forces, $\bar{\mathbf{Q}}$ contains all remaining impressed forces (potential, friction,...), $\boldsymbol{A}^{\boldsymbol{T}}$ is a control matrix, $\mathbf{u}$ is a vector of control forces and $\mathbf{m}$ is a vector of dynamic parameters (moment of inertia, mass, position of the center of mass,...).

For the real mechanism we can write the equations of motion in form

$$\underline{\bar{\mathbf{G}}}\left(\underline{\mathbf{q}},\underline{\mathbf{l}},\underline{\mathbf{m}}\right)\underline{\ddot{\mathbf{q}}}_2 + \underline{\bar{\mathbf{C}}}\left(\underline{\mathbf{q}},\underline{\dot{\mathbf{q}}},\underline{\mathbf{l}},\underline{\mathbf{m}}\right)\underline{\dot{\mathbf{q}}}_2 + \underline{\bar{\mathbf{Q}}}\left(\underline{\mathbf{q}},\underline{\dot{\mathbf{q}}},\underline{\mathbf{l}},\underline{\mathbf{m}},\mathrm{t}\right) = \underline{\mathbf{A}}^T\left(\underline{\mathbf{q}},\underline{\mathbf{l}}\right)\mathbf{u}. \tag{5}$$

A difference between the dynamics of the mechanism and the model is now considered as

$$\Delta\ddot{\mathbf{q}}_2 = \underline{\ddot{\mathbf{q}}}_2 - \ddot{\mathbf{q}}_2. \tag{6}$$

By combining (4) and (5) with simple auxiliary substitution we can write

$$\Delta\ddot{\mathbf{q}}_2 = \underline{\mathbf{f}}\left(\underline{\mathbf{q}},\underline{\dot{\mathbf{q}}},\underline{\mathbf{l}},\underline{\mathbf{m}},\mathrm{t}\right) - \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},\mathbf{l},\mathbf{m},\mathrm{t}) + \left[\underline{\mathbf{B}}\left(\underline{\mathbf{q}},\underline{\mathbf{l}},\underline{\mathbf{m}}\right) - \mathbf{B}(\mathbf{q},\mathbf{l},\mathbf{m})\right]\mathbf{u}. \tag{7}$$

Finally a dynamic model error in terms $\Delta\mathbf{f}$ and $\Delta\mathbf{B}$ is obtained as

$$\ddot{\mathbf{q}}_2 = \mathbf{f} + \Delta\mathbf{f} + [\mathbf{B} + \Delta\mathbf{B}]\mathbf{u}, \tag{8}$$

where

$$\begin{gathered}\Delta\mathbf{f}\left(\underline{\mathbf{q}},\underline{\dot{\mathbf{q}}},\underline{\mathbf{l}},\underline{\mathbf{m}},\mathbf{q},\dot{\mathbf{q}},\mathbf{l},\mathbf{m},\mathrm{t}\right) = \underline{\mathbf{f}}\left(\underline{\mathbf{q}},\underline{\dot{\mathbf{q}}},\underline{\mathbf{l}},\underline{\mathbf{m}},\mathrm{t}\right) - \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},\mathbf{l},\mathbf{m},\mathrm{t}) \\ \Delta\mathbf{B}\left(\underline{\mathbf{q}},\underline{\mathbf{l}},\underline{\mathbf{m}},\mathbf{q},\mathbf{l},\mathbf{m}\right) = \underline{\mathbf{B}}\left(\underline{\mathbf{q}},\underline{\mathbf{l}},\underline{\mathbf{m}}\right) - \mathbf{B}(\mathbf{q},\mathbf{l},\mathbf{m})\end{gathered}\ . \tag{9}$$

These terms $\Delta\mathbf{f}$ and $\Delta\mathbf{B}$ are computed from the nominal parameters $\mathbf{l}$ and $\mathbf{m}$ with respect to the bounded values of their uncertainties $\Delta\mathbf{l}$ and $\Delta\mathbf{m}$. The computational algorithm is based on a sequential combination of $\Delta\mathbf{l}_{\mathrm{i}}$ (i = 1, 2, ..., N) and $\Delta\mathbf{m}_{\mathrm{j}}$ (j = 1, 2, ..., M) from the tolerance areas ($\Delta\mathbf{l}$ and $\Delta\mathbf{m}$). This means that the number of numerical iterations is now $N{\cdot}M$.

4. Example

The proposed numerical method of evaluating of the kinematical ($\Delta \mathbf{q}_f$) and the dynamical ($\Delta \mathbf{f}$, $\Delta \mathbf{B}$) model uncertainties is studied on a planar 4RRR mechanism "Crosshead" (Fig. 2). It is selected a nominal trajectory as a circle centered in the center of the workspace. The geometric parameters deviations are $\Delta \mathbf{l} = \pm 2\ \%\ \mathbf{l}$ (2 % nominal values) and deviations of dynamic parameters are $\Delta \mathbf{m} = \pm 2\ \%\ \mathbf{m}$.

The measured coordinates $\underline{\mathbf{q}}_m$ for 4RRR mechanism are $[\underline{\varphi}_{12}, \underline{\varphi}_{13}, \underline{\varphi}_{14}]$. The position of the end-effector $[x, y, \varphi]$ is calculated by forward kinematics (nominal $\mathbf{l}$). The coordinates of the end-effector are also used as the independent coordinates ($\mathbf{q}_2, \underline{\mathbf{q}}_2$) in the equations of motion (4) $[x, y, \varphi]$ and (5) $[\underline{x}, \underline{y}, \underline{\varphi}]$.

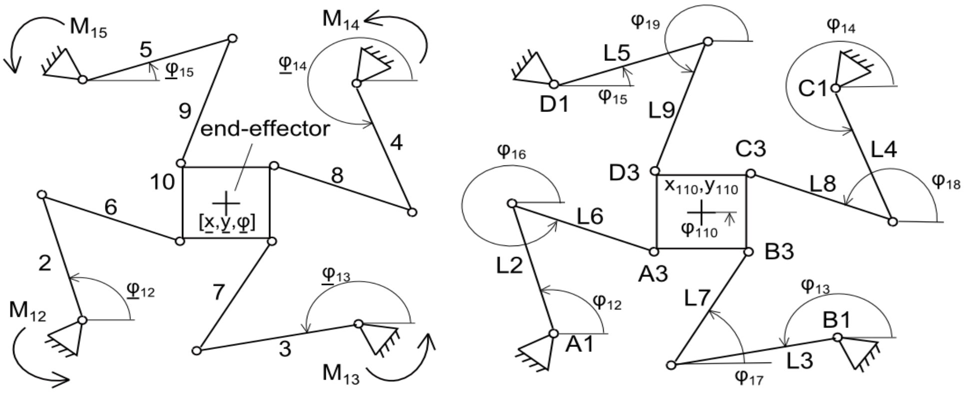

Fig. 2: Mechanism 4RRR with kinematic model ("Crosshead").

The main kinematic model uncertainties ($\Delta x, \Delta y$) are depicted in (Fig. 3). The red line corresponds to the nominal values of the geometric parameters $\mathbf{l}$ and the blue circles are the particular solutions for $\Delta \mathbf{l}_i$.

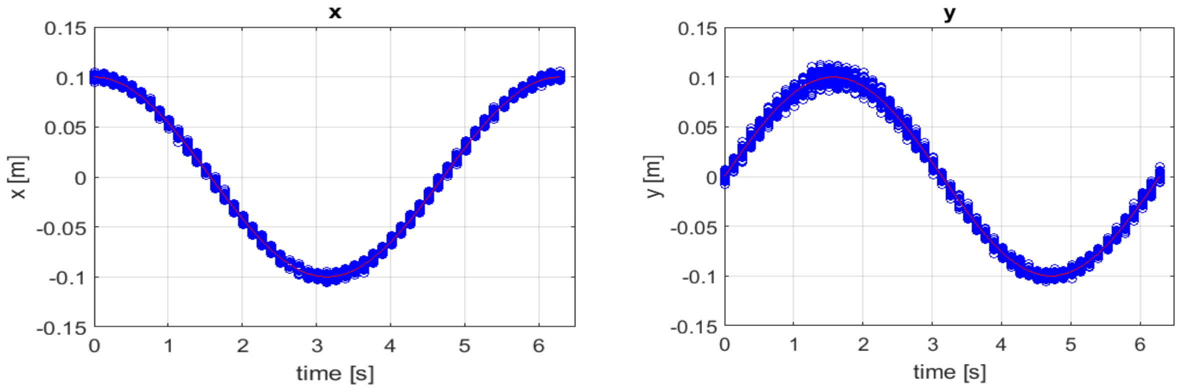

Fig. 3: Kinematic model errors $\Delta \boldsymbol{q}_f$ (Δx, Δy).

Some dynamic model uncertainties ($\Delta f_1, \Delta B_{11}$) are depicted in (Fig. 4). Only the geometric parameters $\Delta \mathbf{l}_i$ are changed here. The calculations showed a much more significant dependence on these parameters.

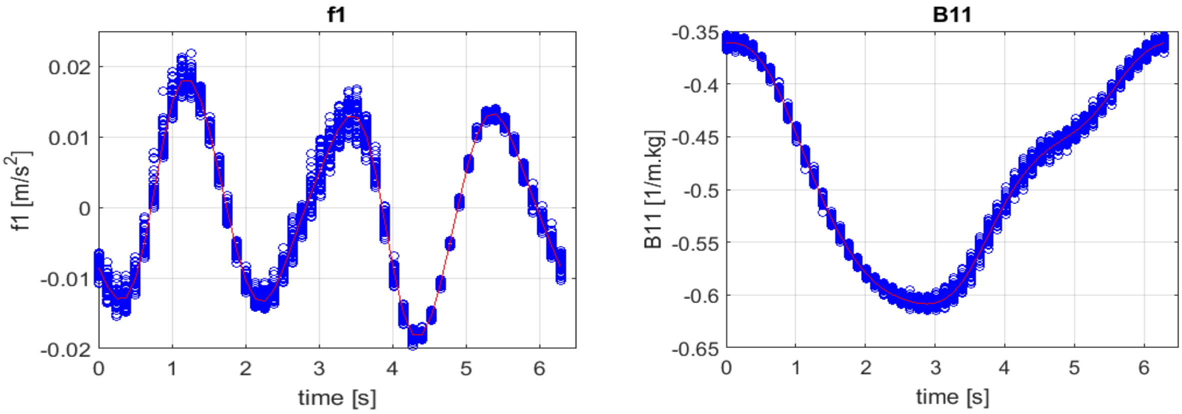

Fig. 4: Dynamic model errors $\Delta \boldsymbol{f}$ (Δf_1), $\Delta \boldsymbol{B}$ (ΔB_{11}).

An example of the mutual influence of the kinematic parameters $\Delta\mathbf{l}$ and the dynamic parameters $\Delta\mathbf{m}$ is depicted in (Fig. 5). Both uncertainties of these parameters are equal ($\Delta\mathbf{l} = \pm 2\,\%\,\mathbf{l}$, $\Delta\mathbf{m} = \pm 2\,\%\,\mathbf{m}$). The red line corresponds to the nominal values $\mathbf{l}$ and $\mathbf{m}$. The blue circles are particular selections $\Delta\mathbf{l}_i$ from $\Delta\mathbf{l}$ and the purple points are a sequential selections $\Delta\mathbf{m}_j$ from $\Delta\mathbf{m}$.

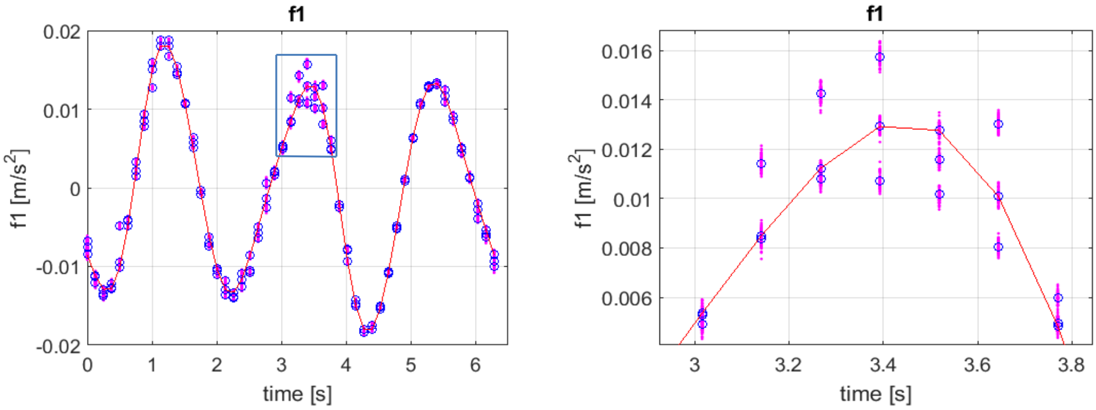

Fig. 5: Influence of the dynamic parameters $\Delta\boldsymbol{m}$ versus the kinematic parameters $\Delta\boldsymbol{l}$.

Finally, all model uncertainties of 4RRR mechanism (kinematic and dynamic) are listed in (Tab. 1).

Tab. 1: 4RRR mechanism – model uncertainties for the nominal trajectory.

Model uncertainties of 4RRR mechanism $\Delta\mathbf{l} = \pm 2\,\%\,\mathbf{l}, \Delta\mathbf{m} = \mathbf{0}, \mathrm{N} = 50, \mathrm{M} = 1(\mathrm{nominal}), \Delta t = \frac{2\pi}{50}$					
Δx_{max}	0.01	[m]	ΔB_{13max}	0.02	$[kg^{-1} \cdot m^{-1}]$
Δy_{max}	0.02	[m]	ΔB_{14max}	0.05	$[kg^{-1} \cdot m^{-1}]$
$\Delta \varphi_{max}$	0.17	[rad]	ΔB_{21max}	0.05	$[kg^{-1} \cdot m^{-1}]$
$\Delta \dot{x}_{max}$	0.005	$[m \cdot s^{-1}]$	ΔB_{22max}	0.02	$[kg^{-1} \cdot m^{-1}]$
$\Delta \dot{y}_{max}$	0.01	$[m \cdot s^{-1}]$	ΔB_{23max}	0.05	$[kg^{-1} \cdot m^{-1}]$
$\Delta \dot{\varphi}_{max}$	0.1	$[rad \cdot s^{-1}]$	ΔB_{24max}	0.02	$[kg^{-1} \cdot m^{-1}]$
Δf_{1max}	0.01	$[m \cdot s^{-2}]$	ΔB_{31max}	0.5	$[rad \cdot kg^{-1} \cdot m^{-2}]$
Δf_{2max}	0.01	$[m \cdot s^{-2}]$	ΔB_{32max}	0.3	$[rad \cdot kg^{-1} \cdot m^{-2}]$
Δf_{3max}	0.1	$[rad \cdot s^{-2}]$	ΔB_{33max}	0.3	$[rad \cdot kg^{-1} \cdot m^{-2}]$
ΔB_{11max}	0.02	$[kg^{-1} \cdot m^{-1}]$	ΔB_{34max}	0.25	$[rad \cdot kg^{-1} \cdot m^{-2}]$
ΔB_{12max}	0.02	$[kg^{-1} \cdot m^{-1}]$			

5. Conclusion

The proposed method of calculation of the model uncertainties needs generally $N{\cdot}M$ numerical iterations. Fortunately the simulation results suggest a reduction to N numerical iterations in the case of a more significant influence of the geometric parameters. The method is generally valid for the nominal trajectory only. To cover the whole workspace of the mechanism it is desirable to augment this method. The more general applicability and the calculation complexity are main directions for the development of this topic. A promising approach is to use the developed issue of tolerance spaces (Stejskal, 1996).

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