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DESIGN OF SUITABLE ASYMPTOTIC COURSES OF METEO-BALLISTIC WEIGHTING FACTOR FUNCTIONS FOR SENSITIVITY ANALYSIS OF PERTURBED PROJECTILE TRAJECTORIES

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Abstract: *This paper is a free continuation of the paper at Engineering Mechanics 2014 by Čech, Jevický and Jedlička. In practice, various relationships are used to sensitivity analysis the effect of meteo conditions on the uncontrolled projectile trajectory, all of which are derived from the corresponding Weighting Factor Functions - WFFs. One of the ways to analyze the influence of the trend of meteo conditions on the projectile trajectory is the use of simplified - asymptotic courses of WFFs. Thus, for example, Weighting Factors - WFs for STANAG 4061 were created in this way. In the article, we deal with the analysis of the properties of one group of asymptotic WFFs, which is suitable for surface-to-surface fire and the effect of wind on the projectile trajectory with altitude up to cc. 10 km (range up to cc. 30 km) and virtual temperature up to altitude cc. 6 km (range up to cc. 20 km). From the given WFFs, the corresponding systems of WFs are easily calculated. At the same time, we present the relations for the calculation of the Reference Height of projectile trajectory, which is given in the tabular firing tables according to the Soviet methodology.*

Keywords: Perturbation of the projectile trajectory, Weighting factor function (curve), Weighting factor, (meteorological) Ballistic elements, Firing tables.

1. Introduction

Calculations of the position of Mean Point of Impact/Burst (MPI) using Tabular Firing Tables (TFT) assume the use of so-called "*ballistic values*" of wind $\mathbf{w}_B = (w_{x,B}, 0, w_{z,B})$, air density ρ_B and virtual temperature $T_{v,B}$ according to NATO STANAG 4061 and 4119 methodology or $\mathbf{w}_B = (w_{x,B}, 0, w_{z,B})$, virtual temperature $T_{v,B}$ and air pressure $p(h_G)$, where h_G is the altitude of the barrel muzzle of the weapon/gun (according to Soviet methodology). To calculate ballistic values ($\mathbf{w}_B$, ρ_B, $T_{v,B}$), the corresponding Weighting Factors - WFs are used, which are simply calculated from the corresponding Weighting Factor Functions - WFFs. According to the Soviet methodology, WFs are not used, but the corresponding Reference Heights Y_R of the projectile trajectory - RHTs are calculated from the given WFFs (Kovalenko and Shevkunov, 1975) and (Cech et al., 2014a). Current ballistic values are presented in special artillery meteorological messages, for example METBKQ (according to STANAG 4061).

WFFs and Green's functions are also special sensitivity functions, without which it is not possible to perform high-quality sensitivity analysis of the influence of meteo conditions on changes in the unguided projectile trajectory.

Complications in the calculation of WFFs are caused by the so-called "*norm effect*". An improved theory of generalized meteo-ballistic WFFs has been published (Cech and Jevicky, 2016 and 2019), which successfully overcomes these problems. This theory was supplemented by an improved theory of generalized RHTs (Cech and Jevicky, 2017).

Another complication (Bliss, 1944), (Molitz and Strobel, 1963) is that the shape of a given WFFs depends on the specific value of the ballistic coefficient c, initial (muzzle) velocity v_0 and angle of departure Θ_0. Instead of the dependence on the angle Θ_0, the dependence on the trajectory vertex (summit) $y_S = f(c, v_0, \Theta_0)$ is more often mentioned.

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Methodologies for generating meteo messages similar to METBKQ assume the dependence of WFs only on the trajectory vertex y_S and ignore the dependence on ballistic coefficient c and initial velocity v_0. This simplifies the whole problem, but at the cost of reducing the accuracy of the calculated ballistic values. In other words, WFs are "averaged" for usual calibers (cc. 30 to 300 mm) and usual initial velocities v_0 (cc. 100 to 1300 m/s). Thus, "averaged" WFFs are used to calculate WFs, which express the trend or asymptotic component of the course of real WFFs (Cech et al., 2014a, b).

For sensitivity analysis, these "averaged" or asymptotic WFFs can be used in the first approximation. Their suitably selected subsets can be advantageously approximated by simple analytical functions.

In our paper we present an approximation of WFFs r(τ, n) - Fig. 1a and r(η, n) - Fig. 2b for surface-to-surface fire, which is suitable for the approximation of real WFFs for windw = (wx, 0, wz,), up to an altitude vertex yS of 10 km, which corresponds to the range up to cc. 30 km, and for virtual temperature T_v up to altitude cc. 6 km, which corresponds to the range up to cc. 20 km.

2. Weighting factor functions approximation

The basic definition or approximation of WFFs r(τ, n) is in the time domain - Fig. 1a. The approximation parameter is the exponent n ≥ 0. From WFF we obtain by deriving the corresponding Green's (impulse response) function g(τ, n) – Fig. 1b (Cech. and Jevicky, 2016 and 2019).

However, for practical calculations, WFFs r(η, n) – Fig. 2b and Green's functions g(η, n) – Fig. 3a in vertical coordinate y domain (y – coordinate domain)with Bliss'notation (Bliss, 1944) are more often used (Cech and Jevicky, 2016 and 2019).

The conversion of functions from time domain to y - coordinate domain is performed using the function t = f (y), which is selected in the first approximation as a result of vacuum parabolic trajectory theory. As an intermediate result we obtain a *two-branched effect function*$r_{(2)}(\eta, n)$ - Fig. 2a. From it, WFFs in y – coordinate domain in Garnier's and Bliss'notations are calculated (Cech and Jevicky, 2016 and 2019).

In our case we get

$$r(\eta, n) = 1 - \frac{1}{2^{n+1}}\left\{\left[1+\sqrt{1-\eta}\right]^{n+1} - \left[1-\sqrt{1-\eta}\right]^{n+1}\right\} \qquad (1)$$

and further

$$\gamma(\eta, n) = \frac{g(\eta,n)}{g(\eta,0)} = \frac{n+1}{2^{n+1}}\left\{\left[1+\sqrt{1-\eta}\right]^{n} + \left[1-\sqrt{1-\eta}\right]^{n}\right\}. \qquad (2)$$

The relation holds for the calculation of ballistic values ($\Delta\mu = \{w_x, w_z, \Delta T_v, \Delta\rho\}$)

$$\Delta\mu_B(n) = \int_0^1 \Delta\mu(\tau)\cdot g(\tau, n)\cdot d\tau = \int_0^1 \Delta\mu(\eta)\cdot g(\eta, n)\cdot d\eta, \qquad (3)$$

where: $\Delta\mu = \mu - \mu_{STD}$– absolute deviation of meteo parameter μ,

μ - measured or model values,

μ_{STD} – standard values.

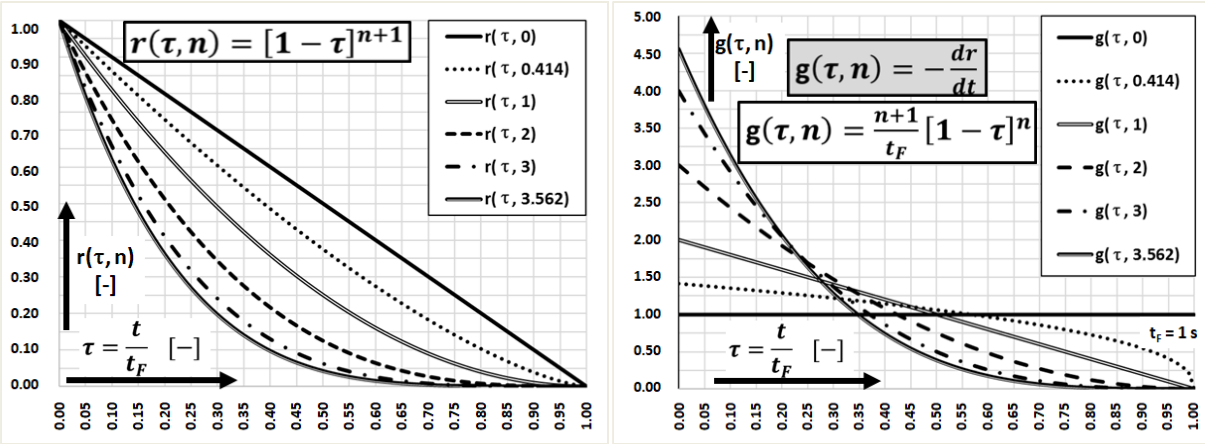
A similar relationship applies to relative deviations $\delta\mu = \Delta\mu/\mu_{STD}$ (Cech and Jevicky, 2016 and 2019).

3. Analysis of relations for WFFs and Green's functions

Eq. (3) represent *Duhamel's convolution integrals*, which can also be interpreted as formulas for calculating the *weighted average* $\Delta\mu_B$ of a given function Δμ using the "weight" function - Green's function g.

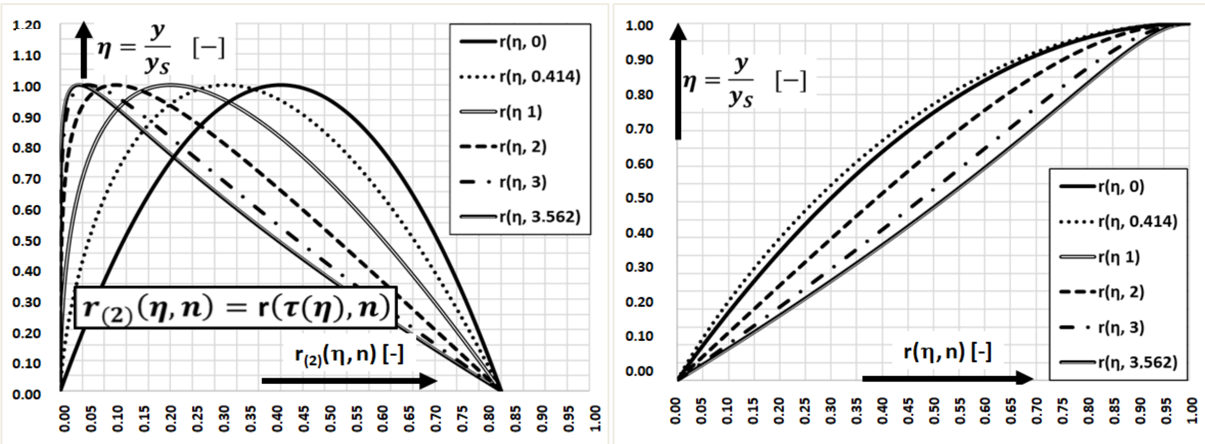
For n = 0 the integral in the time domain (Fig. 1b) represents the calculation of the *arithmetic average* for g(τ, 0) = 1, while in the y - coordinate domain it is already a weighted average.

These relations were used by the French mathematician M. E. Borel to calculate ballistic wind as early as the 1910s (Cranz, 1925). However, *the course g(τ,* 0*) contradicts physical reality, because g(τ)* = 0 *for τ ≥* 1 *must hold.* The real courses are approximated for n > 0.The question arises as to how these approximations have been used successfully for more than 100 years. The answer is paradoxical: *The courses of both WFFs and Green's function for n = 0 and 1 are identical in y - coordinate domain* - Fig. 2b, Fig. 3a, although they have completely different courses in the time domain – Fig. 1. As far as we know, we are the first to publish this revelation here.



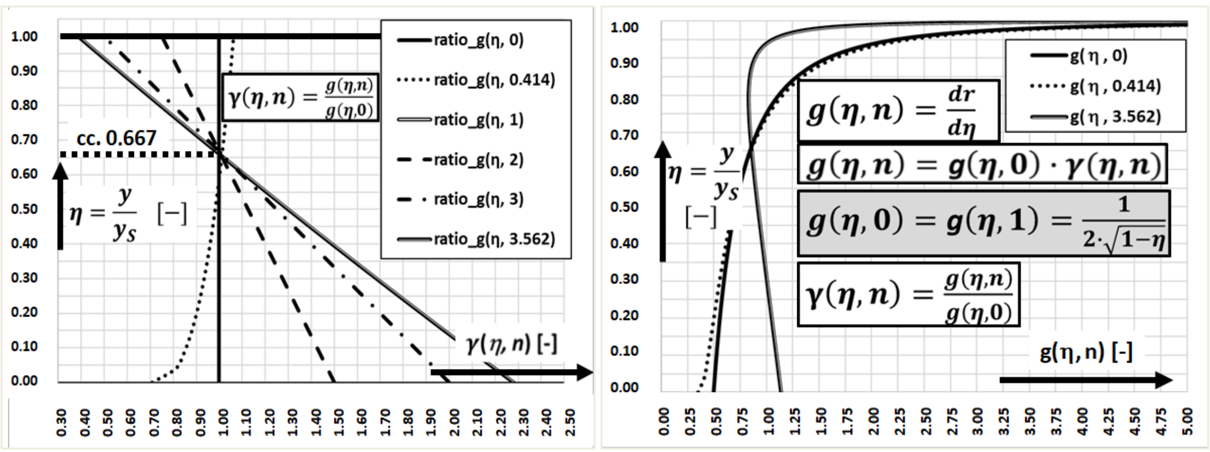
a) b)

Fig. 1: a) Weighting factor functions – WFFs r(τ, n); b) Green's functions g(τ, n) in time domain (t_F – time of flight to the point of fall; chosen for simplicity t_F = 1 s).



a) b)

Fig. 2: a) Two-branched effect functions $r_{(2)}(\eta, n)$; b) Weighting factor functions – WFFs r(η, n) in vertical coordinate y domain (y – coordinate domain).



a) b)

Fig. 3: a) Green's functions g(η, n); b) ratio γ(η, n) in vertical coordinate y domain (y – coordinate domain).

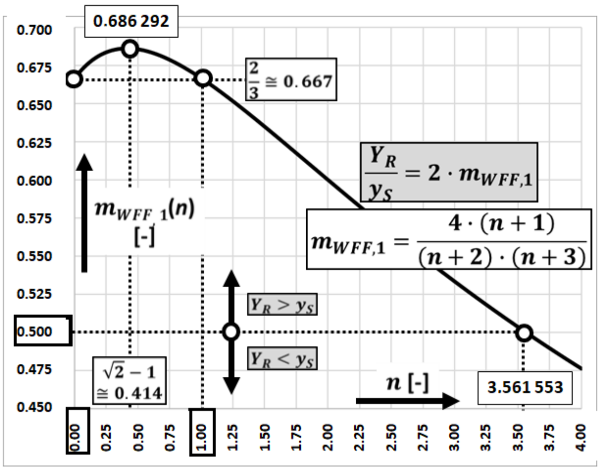


Fig. 4: Moment $m_{WFF,1}(n)$ of the Weighting Factor Functions $r(\eta, n)$.

From Fig. 3a it is clear that Green's functions g(η, n) diverge for η→ 1, closer to Cech V. and Jevicky J. (2019).

From Fig. 3a, b shows that as n (n ≥ 0.414) increases, the weight of a part of the projectile's trajectory near its vertex (η > cc. 0.667) decreases, and then the weight of its part at the ground increases (η < cc. 0.667). As a result, the reference height YR of projectile trajectory also changes – Fig. 4.

4. Conclusions

In the following period, we will gradually publish other suitable asymptotic approximations of WFFs.

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